Complexity and Precision Analysis of DFT Based ENF Extraction Methods

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ABSTRACT
This paper gives a complexity and precision analysis of methods of Discrete Fourier Transform (DFT) based algorithm to extract the Electric Network Frequency (ENF) information of an audio recording in audio authentication. The basic idea of DFT based algorithm is to find specific spectral lines in the frequency domain at the desired frequency point instead of the entire frequency band. Then different line search methods can be employed to search the next desired frequency bin to repeat the spectral line calculation until the hidden ENF information is extracted. The purpose is to improve the accuracy and precision of conventional ENF extraction methods and also to enhance the calculation efficiency. Discrete Fourier Transforms (DFT) based line search algorithm namely ordinary binary search, Fibonacci search and binary quadratic search has been analyzed for accuracy and complexity. 

Keywords:- Audio authentication, digital audio recording, discrete Fourier transform (DFT), electric network frequency (ENF).

I. INTRODUCTION
With the advances in the field of digital signal processing and availability of technological facility digital methods of audio authentication has become a promising field. Analog methods of audio authentication are well established [1] but digital methods face challenges due to ease in alteration of audio recording by means of personal computers and other digital devices. The basic aim of audio authentication is to determine time instant of audio recording, 2) tampers in the recording, and 3) the location of any tamper within the recording if there.

The Electric Network Frequency (ENF) Criterion as proposed by Grigoras in [2]–[4], has been proved as effective and efficient forensic technique in digital audio recording authentication and can detect the tampers like deletion, insertion and substitution in the audio recordings. ENF is basically the frequency of electrical power grid which due to technical reasons fluctuate in close proximity of its nominal value (50/60 Hz) and fluctuation pattern is the same throughout the entire network [2] [5]. When any digital recording equipment (both mains and battery powered) is used for recording it picks the ENF, which ends up as an extra frequency component in the recorded audio file [2],[3] [6]. By band pass filtering the audio signal, the ENF can be isolated and its pattern can be retrieved. There are three main methods for ENF extraction: the time-frequency domain method based on the spectrum analysis, the frequency domain method based on power spectrum analysis by windowed Discrete Fourier Transform (DFT), and the time domain method based on zero crossing measurements of a band-pass filtered signal [2]–[4]. ENF extraction using the frequency domain method has been shown to be valid and effective, but suffers from the ‘uncertainty principle’ of the Fourier Transform and also the low ENF signal energy in the audio recordings [6]. DFT-based algorithm to calculate the specific spectral lines at the desired points improves the accuracy and precision of ENF extraction with the frequency domain method and significantly reduces the algorithm’s complexity.

The paper is organized as follows: Section II gives the basic idea of how to extract the ENF pattern from a digital audio recording. Section III explains the brief background of DFT method of frequency estimation along with its limitations. Section IV gives the brief description about different line search algorithm. Section V is comparative analysis of different methods and section VI is conclusion.

II. ENF PATTERN EXTRACTION FROM DIGITAL AUDIO RECORDING

We have adopted the method presented by Cooper [6] for extracting the ENF pattern from a digital audio recording. We shall cover this method briefly here, since Cooper’s paper offers an excellent and comprehensive description.

The basic steps are:
A. Signal Decimation
Many digital audio recordings are recorded at high sampling frequencies e.g., 44100 Hz. To detect the ENF, which is approximately 50 Hz, much lower sampling frequencies are allowed. The audio file is thus decimated to a sampling frequency of 300 Hz, which significantly reduces computational time.

B. Band Pass Filtering
The frequencies of interest are around 50 Hz, so the decimated audio file is digitally band pass filtered from 49.5 Hz to 50.5 Hz to isolate the ENF.
C. Short Time Fourier Transform (STFT)

In discrete time STFT analysis, a signal is divided into \( J \) partly overlapping frames (figure 1) for which, after windowing and zero-padding, the frequency spectrum is calculated via a Discrete Fourier Transform (DFT). The jump \( H \) (in samples) between frames determines the time resolution of the final ENF pattern, while the amount of overlap \( M - H \) affects its smoothness. In our specific case, we have chosen \( H = 300 \) so that the extracted ENF pattern time resolution equals that of the database – i.e., 1 second. Each frame was windowed with a rectangular window and zero-padded by a factor of 4.

D. Peak Frequency Estimation

For each frequency spectrum, the frequency with maximum amplitude is estimated. As it is unlikely that this ‘peak frequency’ coincides exactly with a DFT frequency bin, quadratic interpolation around the bin with maximum amplitude is performed. The estimated peak frequency is stored as the ENF value for the corresponding frame, so that we end up with an extracted ENF pattern of \( J \) ENF values.

Fig. 1  Division of a signal of length \( N \) into \( J \) partly overlapping frames

III. DISCRETE FOURIER TRANSFORMS (DFT)

Any time sequence/signal is a combination of different sinusoidal components. As frequency is inverse of time, signal can be decomposed into different sinusoidal component of which it is composed of and Fourier transform is one the mathematical tool for doing so.

As most signal of practical interest need to be in discrete domain for better and convenient analysis Discrete Fourier Transforms (DFT) is one of the Fourier transform technique which converts a finite list of equally spaced samples of a time function into the list of coefficient of a finite combination of complex sinusoid, ordered by their frequencies, that has those same sample values.

If \( x(n) \) is a discrete time signal of length \( N \) then DFT converts it to frequency domain signal \( X(k) \) of length \( N \) having coefficient of \( N \) equally spaced frequency.

Mathematically

\[
X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}
\]

Where \( k = 0 \) to \( N-1 \).

In practical application, \( N \) is finite, which causes two changes to the frequency spectrum: 1) the impulse in the frequency spectrum is spread out or broadened; 2) the spectrum is sampled at some specific frequency points and only some specific spectral lines would be obtained by DFT, which is shown in Fig. 2. Fig shows spectral line for infinite and finite sequences. In FFT we do the sampling of frequency spectrum at discrete points so the frequency bin with highest magnitude may or may not be the actual frequency. In fig 2 the frequency bins lying in the main lobe of width \( \Delta f \) are the nearest frequency values to the actual frequency.

![Fig. 2 Spectral line for infinite and finite sequences](image)

IV. ALGORITHMS FOR FREQUENCY ESTIMATION

From previous section we observed that for electric network frequency estimation we need an algorithm by which we can get the frequency having highest magnitude in the main lobe of spectrum. Fig 3 depicts the frequency spectrum for fundamental frequency at 60 Hz. It is obvious that spectrum main lobe can be considered as unimodal function of frequency, so DFT based different line search algorithm can be employed for ENF estimation.

![Fig. 3 Illustration of the relationship among rough ENF, actual ENF, and search band](image)

Some cost effective line search algorithm are

A. Binary Search Algorithm
The binary search method basically divides the search band into two half and looks for the key value in the halves. Retaining the required half, it discards the other half and considering the required half as new search band it starts the same procedure till the key value is obtained.

In our case the algorithm will be
i. Find the amplitude for lower and higher value of frequency band.
ii. Compare the both amplitude.
iii. Higher amplitude frequency and mid frequency of the search band will be the next search band.
iv. Repeat step i till frequency band reach the twice of allowed frequency resolution.
v. Mid frequency of frequency search band is required frequency.

B. Fibonacci Search Algorithm

The Fibonacci search method employs the divide and conquers algorithm using fibonacci numbers. Unlike binary, it divides the search band into two in proportion to golden ratio.

The key is looked into the two parts and search band is reduced accordingly.

In our case the algorithm will be
For \([a, b]\) as frequency band, \(t\) as allowed frequency resolution and \(f(x)\) as amplitude of frequency at \(x\)

i. Calculate lowest Fibonacci number \(F_n\) as \(F_n > (b-a)/t\)

ii. Calculate test point \(x_1 = a + \frac{(F_n - 2/F_n)(b-a)}{2}\)

iii. Calculate \(f(x_1)\) and \(f(x_2)\)

iv. Compare \(f(x_1)\) and \(f(x_2)\)

v. (a) If \(f(x_1) < f(x_2)\)
then the new interval is \([x_1, b]\),
a becomes the previous \(x_1\),
b does not change,
\(x_1\) becomes the previous \(x_2\),
\(n = n - 1\),
Find the new \(x_2\) using the formula in Step ii.
(b) If \(f(x_1) > f(x_2)\)
then the new interval is \([a, x_2]\),
a does not change,
b becomes the previous \(x_2\),
\(x_2\) becomes the previous \(x_1\),
\(n = n - 1\),
Find the new \(x_1\) using the formula in Step ii.
vi. If the length of the new interval from Step iv is less than the \(t\) specified, then stop. Otherwise go back to Step iii.

C. Quadratic Search Algorithm

Binary quadratic search method is similar to binary method where search band is divided into four parts.

V. COMPARATIVE ANALYSIS OF ALGORITHMS

The algorithm complexity for calculation of the frequency spectrum by FFT and DFT is given by \(O(N \log_2 N)\) and \(O(N^2)\) [9]. So the time complexity for DFT based ENF extraction when particular spectral lines are calculated will be \(O(m \log N)\) where \(m\) is no of spectral lines. For \(n\) as no of items in a set on which search has to perform binary search worst case complexity is \(O(\log n)\) while for Quadratic search it is \(O(\log(n/2))\) [7] while Fibonacci search is 11.8% efficient than Binary search[8]. For accuracy of methods Table I shows the value for different frequency as fundamental frequency searched by different methods in absence of noise and Table II shows the value for fundamental frequency 49.94 with different SNR ratio searched by different methods.

<table>
<thead>
<tr>
<th>Line Search Methods</th>
<th>Fundamental Frequency</th>
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<tbody>
<tr>
<td>Binary search</td>
<td>30.416 60.240 89.870 199.860</td>
</tr>
<tr>
<td>Fibonacci search</td>
<td>30.412 60.238 89.871 199.859</td>
</tr>
<tr>
<td>Quadratic search</td>
<td>30.416 60.240 89.869 199.860</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>SNR</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Binary search</td>
<td>49.937 49.943 49.941 49.940</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fibonacci search</td>
<td>49.936 49.943 49.943 49.942</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quadratic search</td>
<td>49.937 49.943 49.941 49.940</td>
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VI. CONCLUSION

From section V we can observe that as per accuracy is concerned Binary search and Quadratic search are same and better than Fibonacci search while in terms of efficiency Quadratic search and Fibonacci search are better than Binary search. So basically there is tradeoff between accuracy and efficiency but overall Quadratic is preferable.
REFERENCES