An Approach to find node Voltages, Branch Currents and Power Losses of a Single and Three Phase Distribution System

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ABSTRACT
The distribution system starts from the distribution substation and ends at the consumer end. It is divided into primary (at relatively high voltage), secondary (at medium voltage) and tertiary (at 400/230 V 3phase/1 phase). Distribution at high voltage is essentially meant for bulk consumers or transfer of power from one place to another for convenience of distribution. In primary and secondary distribution, feeders are used to cater large power from a primary distribution substation to large industrial consumers or to secondary/tertiary substations. In the tertiary stage, distributors are used to cater power to individual consumers from the distribution poles (or tapping points for underground distribution in densely populated urban places). The distributors are tapped at several points. The system may be radial with branches and sub-branches or ring-main. The solution of radial lines is simple but computer iterations are necessary for the same. In this paper an attempt has been made to find out current as well as active and reactive power loss of a single and three phase distribution system by computer programming through nested loop

Keywords:- Single Phase, Three Phase, Power loss, Node Voltage, Branch Current, Distribution System

I. INTRODUCTION
Power flow in a transmission (grid) system is very complex. It involves hundreds of buses and thousands of connecting lines. Some buses are load (P-Q) buses and some buses are generating (P-V) buses. A slack bus is also necessary for balancing the equations. That level of complexity is not generally present in distribution systems. In primary and secondary distribution, feeders are used to cater large power from a primary distribution substation to large industrial consumers or to secondary/tertiary substations. In the tertiary stage, distributors are used to cater power to individual consumers from the distribution poles. The distributors are tapped at several points. The system may be radial with branches and sub-branches or ring-main. The solution of radial lines is simple but computer iterations are necessary for the same.

3-phase distributors cater both 1-phase and 3-phase loads. In spite of our best attempts, it is impossible to balance the load between 3-phases. However, at the outset we may neglect the imbalance in load. The line impedances can be found out from geometry of configuration, using standard formulae. But the load is seldom modeled as impedance.

The load may be modeled as:  a) P-Q load or P- p.f load b) current- p.f load d) P and Q as functions of voltage as well as frequency.

II. LOAD-FLOW STUDY IN GRID SYSTEM-DIFFERENT ALGORITHMS
A no. of different algorithms is used for conducting load flow study in a transmission system viz.

a) Gauss-Siedel iterative method- it is less efficient, takes more time and more no of steps for the convergence. The convergence is linear.

b) Newton-Raphson method- It is more efficient, takes less no of steps and the convergence is quadratic. This is again divided into i) rectangular and ii) polar. The polar method is more tractable and hence popular.

c) Fast decoupled method- It is an approximate method developed from the N-R method. It takes less no of steps and makes the convergence faster.
III. COMPUTATION OF LOSSES

Transmission system losses can be calculated by Newton-Raphson/Gauss-Seidel method but not in the distribution system. Transmission system losses, both active and reactive, are computed by load flow study. Newton-Raphson (rectangular or polar) and Gauss-Seidel methods are methods for load flow analysis. In these methods the losses are computed as:

\[ P_{loss} = \sum P_g - \sum P_i \quad \text{And} \quad Q_{loss} = \sum Q_i - \sum Q_L, \]

where \( \sum P_g \) is the total generation inclusive of the slack bus and \( \sum P_L \) is the sum of the active powers drawn by the loads, \( \sum Q_i \) is the total VAR generation inclusive of the contributions by transmission lines and \( \sum Q_L \) is the sum of reactive powers drawn by the loads. In load low of distribution systems, these figures cannot be obtained. So other methods are to be applied to compute the losses. If currents in each section are known, as well as the line impedance of each section, then we can use:

\[ P_{loss} = \sum I^2 R \quad \text{and} \quad Q_{loss} = \sum I^2 X. \]

IV. BACKWARD / FORWARD ALGORITHM

BW/FW Sweep algorithm is a method frequently used for load flow analysis of radial distribution network. The backward-forward method includes two steps: the backward sweep and the forward sweep. In backward sweep, voltage and currents are computed using KVL and KCL from the farthest node from the source node. In forward sweep, the downstream voltage is calculated starting from source node. The input data of this algorithm is given by node-branch oriented data. Basic data required are, active and reactive powers, nomenclature for sending and receiving nodes, and positive sequence impedance model for all branches. Listed below summarize major steps of the proposed solution algorithm with appropriate equations.

V. NEW EFFICIENT APPROACHES

In a paper by T. Thakur, and J. Dhiman, a new approach to solve the load flow problem in radial distribution networks has been presented. In this approach, the choice of the switches to be opened is based on the calculation of voltage at the buses, real and reactive power flowing through lines, real power losses and voltage deviation, using distribution load flow (DLF) program. As the load flow method was found to give better result than those obtained by some other methods, we have tried the same in this work.
Reactive power loss = 263.94VAR  
% active loss = 4.89; % reactive loss = 33

VIII. PHASE FEEDER

The same procedure is to be followed for 3-phase system. Only difference is to find out phase quantities beforehand. Neglecting the effect of unbalance (with ref. to fig. 1):

Sending end line/phase voltage = 400 V / 230.94 V  
Receiving end load: 5400 +j 3000 W/VAR  
Load p.f. = 0.87416  
Line impedance/phase = (1 +j 3) Ω  
Receiving end line/phase voltage = 360 V / 207.85 V  
Line current = 9.907 A  
Active power loss = 294.44 W  
Reactive power loss = 883.33 VAR  
% active loss = 1.7%  
% reactive loss = 14.3%

IX. 3-PHASE FEEDERS WITH INTERMEDIATE LOAD

Some of the feeders may have a load at an intermediate point. We take such an example. The receiving end voltage is found by similar iteration.

\[
\begin{align*}
V_j & \quad V_r \\
\frac{(R+jX_j)}{2} & \quad \frac{(R+jX_r)}{2} \\
P+q & \quad P+q \\
Q_1 & \quad Q_2 \\
P_1 & \quad P_2 \\
Q_1 & \quad Q_2
\end{align*}
\]

Fig. 2 Feeder with intermediate load  
Sending end voltage = 230 V  
Receiving end load: 1800 +j 800 W/VAR  
Load at intermediate point: 2000 +j 1000 W/VAR  
Frequency = 50 Hz  
Line impedance, 1st stage = (0.5 +j 1) Ω  
Line impedance, 2nd stage = (0.4 +j 0.9) Ω  
Receiving end voltage = 214.33 V  
Voltage at intermediate point = 222.35 V  
Line current, 1st stage = 9.19052 A  
Line current, 2nd stage = 9.19419 A  
Active power loss = 190.33 W  
Reactive power loss = 417.69 VAR  
% active loss = 5; % reactive loss = 23.2

X. DISTRIBUTORS WITH MULTIPLE TAPS

The method can be extended to distributors having multiple loads at several taps. Similar procedure has been adopted. The receiving end voltage has been approximated and then corrected by using iterative method. The results are shown below-the figure is similar to fig. 2 with additional taps in between.

Sending end line/phase voltage = 400/ 230.94 V  
No of poles or taps = 5  
The active and reactive loads at the taps are given below:  
Pole-1: 2000  
Pole-2: 2200  
Pole-3: 1800  
Pole-4: 2400  
Pole-5: 1700  

Sending end line/phase voltage = 400 V / 230.94 V  
No of poles or taps = 5  
The active and reactive loads at the taps are given below:  
Pole-1: 2000  
Pole-2: 2200  
Pole-3: 1800  
Pole-4: 2400  
Pole-5: 1700  

The active & reactive currents, current, and voltage at poles:

<table>
<thead>
<tr>
<th>Pole</th>
<th>Current</th>
<th>Voltage</th>
<th>Active Power</th>
<th>Reactive Power</th>
<th>Load p.f.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sec-1</td>
<td>15.6559</td>
<td>19.5938</td>
<td>19.59383</td>
<td>2.715705</td>
<td>0.7990 lagging</td>
</tr>
<tr>
<td>Pole-1</td>
<td>228.977</td>
<td>-9.516691</td>
<td>4.848498</td>
<td>1.996842</td>
<td>-1.996842</td>
</tr>
<tr>
<td>Sec-2</td>
<td>12.63576</td>
<td>9.313593</td>
<td>6.517913</td>
<td>2.715705</td>
<td>0.7990 lagging</td>
</tr>
<tr>
<td>Pole-2</td>
<td>220.7399</td>
<td>-7.100574</td>
<td>15.81865</td>
<td>8.123491</td>
<td>3.37082</td>
</tr>
<tr>
<td>Sec-3</td>
<td>6.517913</td>
<td>9.313593</td>
<td>4.848498</td>
<td>1.996842</td>
<td>1.996842</td>
</tr>
<tr>
<td>Pole-3</td>
<td>214.6168</td>
<td>-7.100574</td>
<td>15.81865</td>
<td>8.123491</td>
<td>3.37082</td>
</tr>
<tr>
<td>Sec-4</td>
<td>6.517913</td>
<td>6.517913</td>
<td>8.123491</td>
<td>3.37082</td>
<td>3.37082</td>
</tr>
<tr>
<td>Pole-4</td>
<td>210.4041</td>
<td>-7.100574</td>
<td>15.81865</td>
<td>8.123491</td>
<td>3.37082</td>
</tr>
</tbody>
</table>

XI. RADIAL LINES HAVING BRANCHES/SUB-BRANCHES

A little modification is necessary for radial lines, having branches and sub-branches. A specially constructed program has been made to deal with them. The loads have been treated as current loads against given power factor. The program finds out active and reactive power loss of a distributor (3-ph) having a main line and branches. It also finds out the currents in different sections and voltage at the node and ends of the branches. The loads are equivalent current loads with given power factor.

There is a main section and two branches. The main section has 2 poles. The 1st branch has 2 poles and the 2nd. Branch has 2 poles. Line impedance from pole to pole = (0.04 + j 0.02) Ω
The current loads in the main section are:
50 A at p.f. 0.84 lag
45 A at p.f. 0.82 lag

The current loads in the 1st. branch are:
48 A at p.f. 0.85 lag
40 A at p.f. 0.8 lag

The current loads in the 2nd. Branch are:
52 A at p.f. 0.83 lag
44 A at p.f. 0.81 lag

The voltage drop in the in the 1st. branch = 5.3014 V
Resultant current in the 1st. branch = 66 A at p.f. 0.8258 lag

Active power loss in the 1st. branch = 510.99 W
Reactive power loss in the 1st. branch = 255.50 VAR

The voltage drop in the in the 2nd. Branch = 6.1898 V
Resultant current in the 2nd. Branch = 78.8 A at p.f. 0.821 lag

Active power loss in the 2nd. Branch = 737.06 W
Reactive power loss in the 2nd. Branch = 368.53 VAR

The maximum branch voltage drop = 6.189791 V
Sending end line/phase voltage = 400 / 230.94 V
Phase voltage at the end of branch 1 = 203.85 V
Phase voltage at the end of branch 2 = 202.97 V
The current entering into the main line = 270.88 at p.f., 0.8258 lag
Active power loss = 7118.3 W
Reactive power loss = 3559.2 VAR

XII. RECENTLY USED TECHNIQUES FOR LOAD FLOW ANALYSIS OF DISTRIBUTION SYSTEMS

We have straight-forward analytical methods to compute distribution system variables. With increasing complexity, these methods become complex and cumbersome. To obviate these difficulties a no of modern techniques have evolved. A comprehensive treatment of such analytical methods is given below, along with a case-study following the method suggested by T. Thakur, and Jaswanti Dhiman. So we start from the mathematical treatment of their procedure. Symbols have their usual meaning. Voltages at the buses are computed as:

\[ V^{k+1} = V^k - \Delta V^k \]

\[ \Delta V^k \] is the deviation in voltage after two successive iterations
Real power flow: \( P_i = \text{Re}[V_i (V_i - V_j) y_{i,j}] \)
Reactive power flow \( Q_i = \text{Im}[V_i (V_i - V_j) y_{i,j}] \)
Real power loss \[ \sum_{j=1}^{N} (V_{ss}^j - V_j)^2 y_{ss,j} \] - \[ \sum_{j=1}^{N} PD_j \]

\( V_j \) is the voltage at main substation? \( y_{ss,j} \) is the admittance between main substation and bus \( j \) and \( PD_j \) is real power at bus \( j \).

A voltage deviation index (VDI) has been defined by the authors. If the limits are crossed, the program must try to minimize the VDI. This part has been excluded in our study for simplicity.

XIII. DEVELOPMENT OF ALGORITHM

A sample distribution system is shown in fig. 4

The equivalent current injection at the \( k \)th iteration:

\[ i^k_j = i^k_j V^k_j + j \omega \frac{P_j + j Q_j}{V_j^k} \]

Using Kirchhoff's current law, the branch currents are given as:
\( B_1 = I_2 + I_3 + I_4 + I_5 \); \( B_2 = I_4 + I_5 \); \( B_3 = I_4 \); \( B_4 = I_5 \)
In matrix form: \([B] = [BIBC][I]\): The \([BIBC]\) matrix is an upper triangular matrix, the elements are either 1 or 0. The relation between branch currents and bus-voltages: 
\[ V_2 = V_1 - Z_{12} B_1; \quad V_3 = V_2 - Z_{23} B_2 \]
\[ V_4 = V_1 - Z_{14} B_1 - Z_{34} B_4; \quad V_5 = V_3 - Z_{35} B_4 \]
These expressions can be reduced to:
\[ V_4 = V_1 - Z_{14} B_1 - Z_{24} B_2 - Z_{45} B_4; \]
\[ V_5 = V_3 - Z_{34} B_4 \]
Thus the bus voltages are expressed as functions of line parameters, branch currents and the main substation voltage. This leads to an expression like: 
\[ \Delta V = [DLF][I] \]
These \([BIBC]\) and \([BCBV]\) matrices have been developed in terms of the topology of the network. Combining the equations, we get:
\[ \Delta V = [BIBC][BCBV][I] \]
Or \(\Delta V = [DLF][I]\)
The solution for distribution load flow is obtained through following steps:
\[ i_i^k = i_i V_i^{k-1} + j \times j Q_i V_i^{k-1} = \left( \frac{P_i + j Q_i}{V_i^{k-1}} \right) \]
\[ \Delta V_{k+1} = [DLF][I] \]
\[ [V_{k+1}] = [V_k] + [\Delta V_{k+1}] \]

### References

1. EPRI White Paper, “Integrating distributed resources into electric utility distribution system”, Technology Review, December 2001


