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An Approach to find node Voltages, Branch Currents and Power Losses of a Single and Three Phase Distribution System

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ABSTRACT
The distribution system starts from the distribution substation and ends at the consumer end. It is divided into primary (at relatively high voltage), secondary (at medium voltage) and tertiary (at 400/230 V 3phase/1 phase). Distribution at high voltage is essentially meant for bulk consumers or transfer of power from one place to another for convenience of distribution. In primary and secondary distribution, feeders are used to cater large power from a primary distribution substation to large industrial consumers or to secondary/tertiary substations. In the tertiary stage, distributors are used to cater power to individual consumers from the distribution poles (or tapping points for underground distribution in densely populated urban places). The distributors are tapped at several points. The system may be radial with branches and sub-branches or ring-main. The solution of radial lines is simple but computer iterations are necessary for the same. In this paper an attempt has been made to find out current as well as active and reactive power loss of a single and three phase distribution system by computer programming through nested loop
***Keywords*-:** Single Phase, Three Phase, Power loss, Node Voltage, Branch Current, Distribution System

I. INTRODUCTION

Power flow in a transmission (grid) system is very complex. It involves hundreds of buses and thousands of connecting lines. Some buses are load (P-Q) buses and some buses are generating (P-V) buses. A slack bus is also necessary for balancing the equations. That level of complexity is not generally present in distribution systems. In primary and secondary distribution, feeders are used to cater large power from a primary distribution substation to large industrial consumers or to secondary/tertiary substations. In the tertiary stage, distributors are used to cater power to individual consumers from the distribution poles. The distributors are tapped at several points. The system may be radial with branches and sub-branches or ring-main. The solution of radial lines is simple but computer iterations are necessary for the same.

3-phase distributors cater both 1-phase and 3-phase loads. In spite of our best attempts, it is impossible to balance the load between 3-phases. However, at the outset we may neglect the imbalance in load. The line impedances can be found out from geometry of configuration, using standard formulae. But the load is seldom modeled as impedance.

The load may be modeled as: a) P-Q load or P- p.f load b) current- p.f load d) P and Q as functions of voltage as well as frequency.

II. LOAD-FLOW STUDY IN GRID SYSTEM-DIFFERENT ALGORITHMS

A no. of different algorithms is used for conducting load flow study in a transmission system viz.

a) Gauss-Siedel iterative method- it is less efficient, takes more time and more no of steps for the convergence. The convergence is linear.

b) Newton-Raphson method- It is more efficient, takes less no of steps and the convergence is quadratic. This is again divided into i) rectangular and ii) polar. The polar method is more tractable and hence popular.

c) Fast decoupled method- It is an approximate method developed from the N-R method. It takes less no of steps and makes the convergence faster.

III. COMPUTATION OF LOSSES

Transmission system losses can be calculated by Newton-Raphson / Gauss-Seidel method but not in the distribution system. Transmission system losses, both active and reactive, are computed by load flow study. Newton-Raphson (rectangular or polar) and Gauss-Seidel methods are methods for load flow analysis. In these methods the losses are computed as:

$P_{loss} = \sum P_G - \sum P_L$ And $Q_{loss} = \sum Q_G - \sum Q_L$, where $\sum P_G$ is the total generation inclusive of the slack bus and $\sum P_L$ is the sum of the active powers drawn by the loads, $\sum Q_G$ is the total VAR generation inclusive of the contributions by transmission lines and $\sum Q_L$ is the sum of reactive powers drawn by the loads. In load low of distribution systems, these figures cannot be obtained. So other methods are to be applied to compute the losses. If currents in each section are known, as well as the line impedance of each section, then we can use: $P_{loss} = \sum I^2 r$; $Q_{loss} = \sum I^2 x$

IV. BACKWARD / FORWARD ALGORITHM

BW/FW Sweep algorithm is a method frequently used for load flow analysis of radial distribution network. The backward-forward method includes two steps: the backward sweep and the forward sweep. In backward sweep, voltage and currents are computed using KVL and KCL from the farthest node from the source node. In forward sweep, the downstream voltage is calculated starting from source node. The input data of this algorithm is given by node-branch oriented data. Basic data required are, active and reactive powers, nomenclature for sending and receiving nodes, and positive sequence impedance model for all branches. Listed below summarize major steps of the proposed solution algorithm with appropriate equations.

V. NEW EFFICIENT APPROACHES

In a paper by T. Thakur, and J. Dhiman, a new approach to solve the load flow problem in radial distribution networks has been presented. In this approach, the choice of the switches to be opened is based on the calculation of voltage at the buses, real and reactive power flowing through lines, real power losses and voltage deviation, using distribution load flow (DLF) program. As the load flow method was found to give better result than those obtained by some other methods, we have tried the same in this work.

VI. MODELING

The analysis depends on the model we choose. The last one is possibly the best for its higher accuracy, but it is complex. In normal load-flow study loads are considered as P-Q (though P-Q are not constants, they depend on voltage and freq.). Even against this simplification, no closed form solution is available for a feeder or distributor as the receiving end voltage is unknown. At first, we consider a feeder with P,Q load at its end. Loads may also be treated as current loads, which make the analysis simpler.

VII. SINGLE-PHASE FEEDER

A program has been made to find out the current, active and reactive power loss of a feeder catering a P-Q load. The algorithm is given below:

1. Read R.E. P + j. Q, S.E. voltage (230V), line impedance: R+ j.X_L, convergence const. E
2. Estimate R.E. voltage: V_R = 0.9 V_S.
3. Find out current: I_R = (P-j.Q)/V_R
4. Find out p.f. angle: $\phi = \tan^{-1}(Q/P)$
5. Find out approx. S.E. voltage: $V_{Sa} = V_R + I_R(Rcos\phi + X_L \sin\phi)$
6. If abs |(V_S –V_{SA})| < E, then go to
7. V_R =V_R .V_S/V_{SA} :
8. Go to 3
9. Find out active/reactive power loss $P_{loss} = I_R^2 R; Q_{loss} = I_R^2 X_L$
10. Print out results
11. Stop
12. End

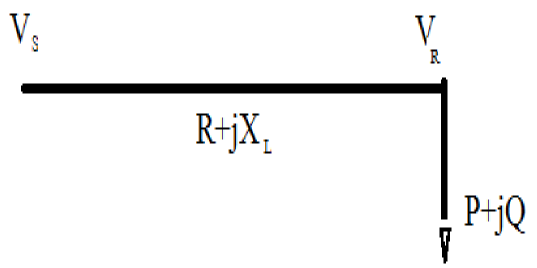

Fig. 1 Single phase or 3-phase balanced feeder

The results are given against hypothetical data:
Sending end voltage= 230 V
Receiving end load, W/VAR: 1800 +j 800
Load p.f. = 0.9138
Line impedance =(1 +j 3) Ω
Receiving end voltage= 210 V
Line current= 9.38 A
Active power loss= 87.98 W

Reactive power loss = 263.94VAR
% active loss = 4.89; % reactive loss = 33

VIII. PHASE FEEDER

The same procedure is to be followed for 3-phase system. Only difference is to find out phase quantities beforehand. Neglecting the effect of unbalance (with ref. to fig. 1):
Sending end line/phase voltage = 400 V/ 230.94 V
Receiving end load: 5400 +j 3000 W/VAR
Load p.f. = 0.87416
Line impedance/phase=(1 +j 3) Ω
Receiving end line/phase voltage = 360 V/ 207.85 V
Line current= 9.907 A
Active power loss = 294.44 W → 1.7%
Reactive power loss = 883.33VAR → 14.3 %

IX. 3-PHASE FEEDERS WITH INTERMEDIATE LOAD

Some of the feeders may have a load at an intermediate point. We take such an example. The receiving end voltage is found by similar iteration.

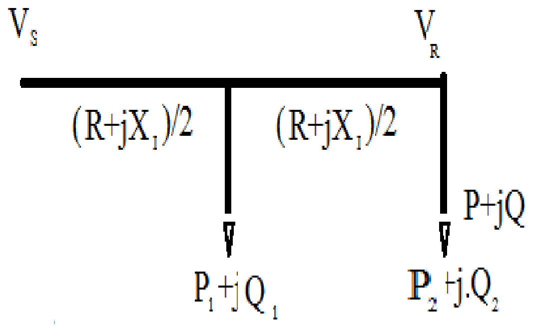

Fig. 2 Feeder with intermediate load
Sending end voltage= 230 V
Receiving end load: 1800 +j 800 W/VAR
Load at intermediate point: 2000 +j 1000 W/VAR
Frequency= 50 Hz.
Line impedance,1st stage=(0.5 +j 1)Ω
Line impedance, 2nd stage=(0 .4 +j 0.9) Ω
Receiving end voltage = 214.33 V
Voltage at intermediate point= 222.35 V
Line current, 1st. stage= 9.19052 A
Line current, 2nd.Stage= 19.2419 A
Active power loss= 190.33 W
Reactive power loss= 417.69 VAR
% active loss = 5; % reactive loss = 23.2

X. DISTRIBUTORS WITH MULTIPLE TAPS

The method can be extended to distributors having multiple loads at several taps. Similar procedure has been adopted. The receiving end voltage has been approximated and then corrected by using iterative method. The results are shown below- the figure is similar to fig. 2 with additional taps in between.
Sending end line/phase voltage= 400/ 230.94
No of poles or taps= 5
The active and reactive loads at the taps are given below:

Pole-1	2000	1500
Pole-2	2200	1600
Pole-3	1800	1450
Pole-4	2400	1800
Pole-5	1700	1250

The active& reactive currents, current, and voltage at poles:

Sec-1	15.6559	-11.7818	
	19.59383	SE-	230.9401
Sec-2	12.63576	-9.516691	15.81865
Pole-1	228.977		
Sec-3	9.313593	-7.100574	11.71158
Pole-2	220.7399		
Sec-4	6.517913	-4.848498	8.123491
Pole-3	214.6168		
Sec-5	2.715705	-1.996842	3.37082
Pole-4	210.4041		

Receiving end line/phase voltage (pole-5)= 361.41/ 208.66 V
Average Load p.f. = 0.7990 lagging.
Line impedance from pole to pole= 0.2 +j 0.6 Ω
Line current= 15.6 6 A
Active power loss = 509.2 W
Reactive power loss= 1528 VAR

XI. RADIAL LINES HAVING BRANCHES/SUB-BRANCHES

A little modification is necessary for radial lines, having branches and sub-branches. A specially constructed program has been made to deal with them. The loads have been treated as current loads against given power factor.
The program finds out active and reactive power loss of a distributor (3-ph) having a main line and branches. It also finds out the currents in different sections and voltage at the node and ends of the branches. The loads are equivalent current loads with given power factor.
There is a main section and two branches. The main section has 2 poles. The 1st branch has 2 poles and the 2nd. Branch has 2 poles. Line impedance from pole to pole = (0.04 + j 0.02) Ω

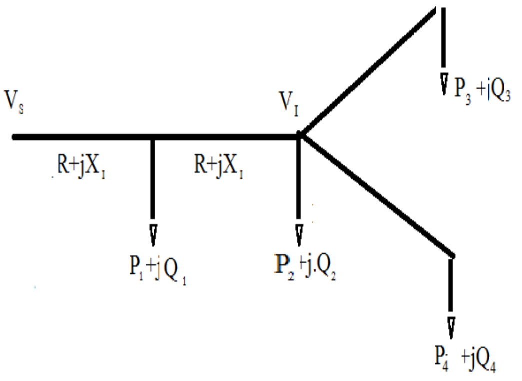

Fig. 3 A radial line with branches

The current loads in the main section are:
50 A at p.f. 0.84 lag
45 A at p.f. 0.82 lag
The current loads in the 1st. branch are:
48 A at p.f. 0.85 lag
40 A at p.f. 0.8 lag
The current loads in the 2nd. Branch are:
52 A at p.f. 0.83 lag
44 A at p.f. 0.81 lag
The voltage drop in the in the 1st. branch = 5.3014 V
Resultant current in the 1st. branch = 66 A at p.f. 0.8258 lag
The voltage drop in the in the 2nd. Branch = 6.1898 V
Resultant current in the 2nd. Branch = 78.8 A at p.f. 0.821 lag
Active power loss in the 1st. branch = 510.99 W
Reactive power loss in the 1st. branch = 255.50 VAR
Active power loss in the 2nd. Branch = 737.06 W
Reactive power loss in the 2nd. Branch = 368.53 VAR
The maximum branch voltage drop = 6.189791 V
The voltage drop in the main section = 21.784 V
Sending end line/phase voltage = 400 / 230.94 V
Phase voltage at the junction of branches = 209.16 V
Phase voltage at the end of branch-1 = 203.85 V
Phase voltage at the end of branch-2= 202.97 V
The current entering into the main line = 270.88 at p.f., 0.8258 lag
Active power loss = 7118.3 W
Reactive power loss = 3559.2 VAR

XII. RECENTLY USED TECHNIQUES FOR LOAD FLOW ANALYSIS OF DISTRIBUTION SYSTEMS

We have straight-forward analytical methods to compute distribution system variables. With increasing complexity, these methods become complex and cumbersome. To obviate these difficulties a no of modern techniques have evolved. A comprehensive treatment of such analytical methods is given below, along with a case-study following the method suggested by T. Thakur, and Jaswanti Dhiman. So we start from the mathematical treatment of their procedure. Symbols have their usual meaning.

Voltages at the buses are computed as:

$$V^{k+1} = V^k - \Delta V^k$$

ΔV^k Is the deviation in voltage after two successive iterations

Real power flow: $P_{ij} = \mathrm{Real}[V_i\{(V_i - V_j)y_{ij}\}^*]$

Reactive power flow $Q_{ij} = \mathrm{Imag}[V_i\{(V_i - V_j)y_{ij}\}^*]$

Real power loss $\{V_{ss}\sum_{j\varepsilon ss}[(V_{ss} - V_j)y_{ss,j}]^* - \sum_{j=1}^{N} PD_j\}$

V_{ss} Is the voltage at main substation? $y_{ss,j}$ is the admittance between main substation and bus j and PD_j is real power at bus j.

A voltage deviation index (VDI) has been defined by the authors. If the limits are crossed, the program must try to minimize the VDI. This part has been excluded in our study for simplicity.

XIII. DEVELOPMENT OF ALGORITHM

A sample distribution system is shown in fig. 4

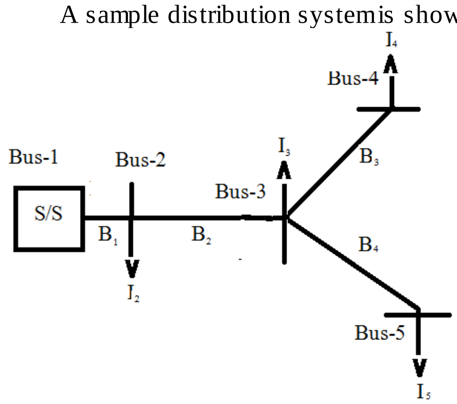

Fig. 4. A sample distribution system with current injection model

For bus i , the complex load is given as: $S_i = (P_i + jQ_i),\ i = 1,....., N$ 5

The equivalent current injection at k^{th} iteration: $i_i^k = i_i^r V_i^k + j.i_i^i V_i^k = \left(\frac{P_i + jQ_i}{V_i^k}\right)$ 6

Using Kirchhoff's current law, the branch currents are given as:

$B_1 = I_2 + I_3 + I_4 + I_5$; $B_2 = I_4 + I_5$; $B_3 = I_4$; $B_4 = I_5$

In matrix form: $\mathbf{[B]=[BIBC][I]}$; The [BIBC] matrix is an upper triangular matrix, the elements are either 1 or 0.
The relation between branch currents and bus-voltages: $V_2 = V_1 - Z_{12}B_1$; $V_3 = V_2 - Z_{23}B_2$
$V_4 = V_3 - Z_{34}B_3$; $V_5 = V_3 - Z_{35}B_4$
These expressions can be reduced to:

$$V_4 = V_1 - Z_{12}B_1 - Z_{23}B_2 - Z_{34}B_4 \quad ;$$

$$V_5 = V_1 - Z_{12}B_1 - Z_{23}B_2 - Z_{35}B_5$$

Thus the bus voltages are expressed as functions of line parameters, branch currents and the main substation voltage. This leads to an expression like: $Delta\,\mathbf{V} = \mathbf{[BCBV][V]}$

These [BIBC] and [BCBV] matrices have been developed in terms of the topology of the network. Combining the equations, we get:

$$\mathbf{\Delta V = [BIBC][BCBV][I]}$$

Or $\mathbf{\Delta V = [DLF][I]}$

The solution for distribution load flow is obtained through following steps:

$$i_i^k = i_i^r V_i^k + j.i_i^i V_i^k = \left(\frac{P_i + jQ_i}{V_i^k}\right)$$

$$\mathbf{\Delta V^{k+1} = [DLF][I^k]}$$

$$\mathbf{[V^{k+1}] = [V^0] + [\Delta V^{k+1}]}$$

The time consuming L-U decomposition and forward backward substitution of the Jacobian matrix or the Y-admittance matrix, required in Newton-Rapson and Gauss implicit Z matrix algorithms, are not necessary in this method. The arithmetic operation, number for LU factorization is approximately proportional to N^3. For a large value of N, the LU factorization will occupy a large portion of the computational time. Therefore if the LUfactorization can be avoided, the load flow method can save computation time to a large extent. The algorithm for this method is given below:

1. Start: read systemdata
2. Form the BIBC and BCBV matrices
3. Multiply them and get the DLF matrix.
4. Calculate change in voltages.
5. Set iteration count k=3 or 4
6. Calculate new values of bus voltages.
7. If count<=k goto step 4
8. Find bus voltages
9. Stop
10. End

For the given example, the matrix [DLF] = [BIBC][BCBV] is being found out.

The upper triangular matrix [BIBC] is given below:

1	1	1	1
0	0	1	1
0	0	0	1
0	0	0	1

The lower triangular matrix [BCBV] is given below:

0.1+ j0.06	0	0	0
0.1+ j0.06	0.1+ j0.06	0	0
0.1+ j0.06	0.1+ j0.06	0.1+ j0.06	0
0.1+ j0.06	0.1+ j0.06	0	0.1+ j0.06

MATRIX [DLF] = [BIBC][BCBV]
0.4+ j0.240.3+ j0.180.1+ j0.06
0.1+ j0.06
0.2+ j0.120.2+ j0.120.1+ j0.060.1+ j0.06
0.1+ j0.060.1+ j0.06 0 0.1+ j0.06
0.1+ j0.060.1+ j0.06 0 0.1+
j0.06

Now, we are in a position to find out the bus voltages using the algorithm.

XIV. CONCLUSION

The methods developed for power flow studies in a distribution system are difficult to understand and apply e.g. forward-backward method. A method developed and reported by T. Thakur, and Jaswanti Dhimanhas been used in our case-study. The approach leads to much complexity even for the simplest example chosen by us. However, computer-based method has been developed for its application to systems. But more stress has been given in developing programs for solution of the branch currents and node voltages using classical methods of circuit analysis. Various example systems have been solved- a single phase and a three-phase feeder with load at their terminals, distribution systems with one or more loads at intermediate points and radial systems with branches and sub-branches. As the loads are of P-Q type mostly (not equivalent current loads), the system becomes non-linear. So an iterative process has been used to compute the node voltages and currents. The algorithm has been given.
A program has been developed for finding out the capacitor support required to improve the p.f. and as such to reduce the voltage drop as well as the reactive power losses.

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