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Time-Frequency Coherence of Multichannel EEG Signals: Synchrosqueezing Transform Based Analysis

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ABSTRACT

This paper presents a novel method of implementing time-frequency coherence between electrophysiological signals for brain-computer interfacing (BCI) paradigm. The neural synchronization mostly depends on both time and frequency. The time-frequency coherence is used to measure neural interdependencies. The short-time Fourier transform (STFT) and wavelet transform are generally used to measure the time-frequency coherence. The limitations of these approaches are resolved using synchrosqueezing transform (SST). Due to data adaptability and frequency reassignment properties, the SST produces a well-defined time-frequency representation. Time-frequency coherence between two synthetic signals and real electroencephalography (EEG) data are illustrated based on the STFT and SST. The experimental results show that the SST based time-frequency coherence executes enhanced result than STFT based approach.

***Keywords:*- STFT, BCI, SST**

I. INTRODUCTION

Electroencephalography (EEG) is cost effective and easier way to implement brain computer interface (BCI). It is captured by spatially distributed EEG sensors of the scalp. The connectivity of different parts of brain is an interesting study to the BCI research community. Recently, the measure of coherence between the signals obtained by different sensors is quantified by coherence analysis [1, 2, 3]. It is usually implemented by spectral estimation with Fourier or wavelet transform. The neural data is usually no stationary in nature and hence it is a great challenge to implement coherence based analysis. The short time Fourier transform (STFT) is considered to solve such problem, it is not entirely resolved due to the following reasons: i) within each short-time period the stationary of neural data cannot be assured, ii) the resolution of time frequency representation is restricted by Heisenberg uncertainty principle [4]. Although wavelet transform is considered as data adaptive signal analysis method, it uses basis function called mother wavelet for signal decomposition and faces time-frequency resolution problem i.e. lower frequency resolution at high frequencies and higher at low frequencies. Wavelet analysis also depends on the selection of mother wavelet. The arbitrary selection of mother wavelet without matching with the analyzing signal is the cause of erroneous and non-reversible Decomposition. The synchrosqueezing transform (SST) [9, 10] is one of the techniques based on the continuous wavelet transform (CWT) that generates highly localized time-frequency representations of nonlinear and no stationary signals. Synchrosqueezing transform overcomes the limitations of linear projection based time-frequency algorithms, such as the short-time Fourier transforms (STFT) and continuous wavelet transforms. The synchrosqueezing transform reassigns the energies of STFT and CWT, such that the resulting energies of coefficients are concentrated around the instantaneous frequency curves of the modulated oscillations [11]. The frequency reassignment method in time-frequency representation [5, 6, 7] develops the meaningful localization of signal components in time-frequency space [8].

In this paper, the TF representation of EEG signals is implemented by SST and then TF coherence between two synthetic signals. The similar analysis is performed with the short-time Fourier transform in place of SST. After validating of the TF coherence paradigm with synthetic signals, the method is applied to real electroencephalography (EEG). It is clearly observed that in both synthetic and real data the SST based TF coherence performs better than STFT based method.

The paper is organized as follows–Section 2 discusses time-frequency representation methods including STFT and SST, the coherence in TF domain is explained in section 3, the experimental results are illustrated in section 4 and the section 5 includes some concluding remarks.

II. TIME-FREQUENCY REPRESENTATION

Time-frequency representation (TFR) of any signal describes the energy as a function of both time and frequency. It maps a one dimensional signal of time, *S(t)*, into a two dimensional function of time and frequency, $T_S(t, f)$. The value of the TFR surface provides idea as to which spectral components are present at what time. The TFR is useful to analyse and synthesize non-stationary or time-varying signals.

A. Short Time Fourier Transform

Short-Time Fourier Transform (STFT) is a time-frequency analysis technique suited to non-stationary signals. The STFT provides information about changes in frequency over time. It represents a sort of compromise between the time and frequency of a signal. Also, it gives some information about both when and at what frequencies a signal event occurs. During STFT, the signal is separated into small portions, where these portions of the signal can be assumed to be stationary. For this purpose, a window function *w* is chosen. The width of this window must be equal to the portion of the signal where its stationarity is valid. The STFT for a non-stationary signal *y(t)* is defined as

$$\Psi(t, f) = \int_{-\infty}^{\infty} [y(t).w^*(t-t')].e^{-2\pi ft}dt \qquad (1)$$

Where * is the complex conjugate, *w(t)* is the window function. The STFT of the signal is the Fourier transform of the signal multiplied by a window function. The signal events are localized in both time and frequency scales in time-frequency space. The time localization clearly identifies signal events which manifest during a short time interval. On the other hand, the frequency localization means to identify the signal components which are concentrated at particular Fourier frequencies, such as sinusoids. In order to better measure a signal at a particular time and frequency, it is natural to desire that the temporal and spectral resolution Δt and Δk respectively be as narrow as possible. If the waveform is well-localized in both of time and frequency, then $\Delta t \Delta k$ will be small. In STFT the selections of Δt and Δk are not independent, which means that the scaling cannot be increased simultaneously both time and frequency resolution [20]. There is a trade-off of the selection of the time and frequency resolution i.e. $\Delta t \Delta k \geq 0.5$. If the window function $w(t)$ is chosen to have good time resolution (smaller Δt, then its frequency resolution must be deteriorated larger Δk), or vice versa. The use of STFT can not provide the desired time-frequency resolution.

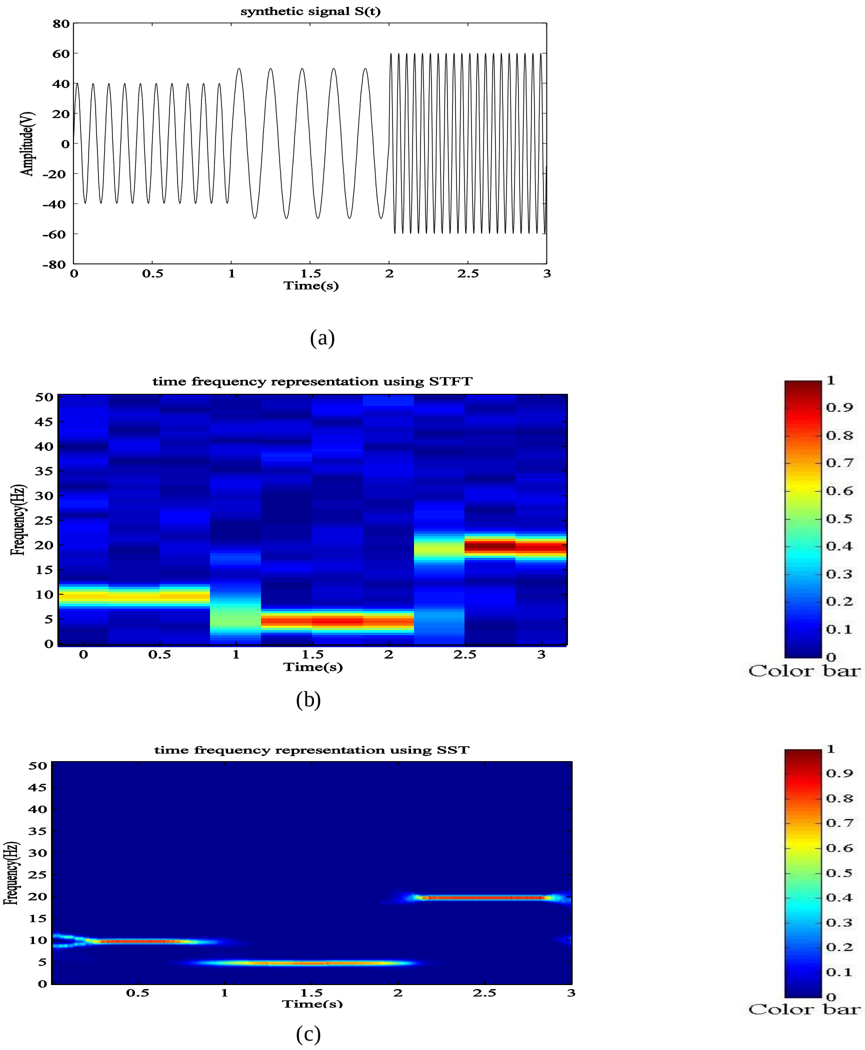

Figure 1: Synthetic signal *S(t)* and its TF representation (a) the synthetic signals *S(t)* with three components, (b) TF representation using STFT and (c) TF representation using SST.

To show time-frequency representation we create a noise free synthetic signal *S(t)* as the horizontal concatenation of the following three components:

$$s_1(t) = 40 * \sin(2 * \pi * 10 * t)$$
$$s_2(t) = 50 * \sin(2 * \pi * 5 * t)$$
$$s_3(t) = 60 * \sin(2 * \pi * 20 * t)$$

The synthetic signal *S(t)* as shown in Fig 1 (a) has three constant harmonics of 10 Hz ($s_1(t)$) from 0 to 1 s, the 5 Hz ($s_2(t)$) from 1 to 2 s and the 20 Hz ($s_3(t)$) from 2 to 3 s. The TF representation using STFT is shown in Fig 1 (b). The sampling frequency of the *S(t)* is 500 Hz. In the STFT a Hamming window of length 256 and 50% overlap is used. The STFT is able to recognize the three components but with a poor resolution.

B. Synchrosqueezing Transform

Synchrosqueezing Transform (SST) is a method functional to the Continuous Wavelet Transform (CWT). The SST is used to make localized TF representations of non-stationary signals. The CWT is capable to create a TF representation of a signal that shows very good TF localization. The SST based TF representation taking the synthetic signal *S(t)* is shown in Fig 1(c). For the SST a bump mother wavelet is used and the discretization of the scales of CWT is 32. The TF representation as shown in Fig 1 shows that the SST based TF representation gives sharper representation of the instantaneous frequencies than the representation based on STFT. The CWT algorithm recognizes oscillatory components of a signal through a series of time-frequency filters known as wavelets. To separate a continuous-time function into wavelets the CWT is used. A mother wavelet $\Phi(t)$ is a finite oscillatory function which convolved with a signal $s(t)$ in the following form

$$Z(p,q)=\frac{1}{|p|^{1/2}}\int_{-\infty}^{\infty}\Phi(\frac{t-q}{p})s(t)dt \tag{2}$$

Where $Z(p, q)$ is the wavelet coefficients for each scale-time pair (p,q). A set of bandpass filter produces the wavelet coefficients. The signal property depends on the scale factor p. The presentation of the signal is more complete when the scale factor is low. On the other hand, high scale factor expands the signal and hence its presentation shows less detail.The scale factors can changes the band width of the bandpass filters. Consequently, the energy of the wavelet transform of a signal will be increased and at original frequency ω_r it will be spread out around the scale factor $p_r = \omega_\varphi/\omega_r$, where ω_φ is the center frequency of a wavelet. Therefore, the original frequency ω_r and the estimated frequency in those scales are same. As a result, for each scale-time pair (p,q) the instantaneous frequency $\omega_s(p,q)$ can be estimated as

$$\omega_s(p,q)=-iZ(p,q)^{-1}\frac{\partial Z(p,q)}{\partial q} \tag{3}$$

The TF representation maps the information from the time-scale plane to the time-frequency plane. In the synchrosqueezing operation, every point (q,p) is converted to $(q,\omega_s(p,q))$ [9]. Because p and q are discrete values, we can have a scaling step $\Delta p_k = p_{k-1}-p_k$ for any p_k where $\omega_s(p,q)$ is computed. During mapping from the time-scale plane to the time-frequency plane $(q,p)\rightarrow(q,w_{inst}(p,q))$, the SST $\Gamma(\omega_l,q)$ is calculated [11] only at the centers ω_l of the frequency range $[\omega_l-\Delta\omega/2,\omega_l+\Delta\omega/2]$, with $\Delta\omega=\omega_l-\omega_{l-1}$:

$$\Gamma(\omega_l,q)=\sum_{p_k:|\omega_s(p_k,q)-\omega_l|\le\Delta\omega/2} Z(p_k,q)p^{-3/2}\Delta p_k \tag{4}$$

The equation (4) shows that the TF representation of the signal is synchrosqueezed along the frequency (or scale) axis only [12]. In the SST, the coefficients of the CWT are reallocated to get a concentrated image over the time-frequency plane, from which the instantaneous frequencies are then extracted [13].

III. TIME-FREQUENCY COHERENCE ANALYSIS

The Time-Frequency (TF) coherence is a measure used to observe the linear correlation between two signals or data sets. In brain computer interfacing motor imagery paradigm the synchronization of neural activity has been measured using the TF coherence. Consider two stationary signals $x(t)$ and $y(t)$. For the signals the standard coherence function is defined as [14]:

$$|C_{x,y}(f)|=\frac{|J_{x,y}(f)|}{\sqrt{J_{x,x}(f)J_{y,y}(f)}} \tag{5}$$

Where $J_{x,y}(f)$ is the cross spectrum between signal x and y, and $J_{x,x}(f)$ and $J_{y,y}(f)$ are the auto spectrums of the signal x and y respectively. The standard coherence function is not enough for EEG like non-stationary signals. Alternatively, a time-frequency extension approach measures the linear correlation between two signals in time-frequency plane [15]. The TF coherence of two non-stationary signals x and y is defined as

$$|C_{x,y}(t,f)|=\frac{|J_{x,y}(t,f)|}{\sqrt{J_{x,x}(t,f)J_{y,y}(t,f)}} \tag{6}$$

where $t=1,2,\dots,T$; signal partitioned into T segments and $f=1,2,\dots,F$; is the discrete frequency. The cross and auto spectrums are calculated as

$$\begin{gathered}J_{x,y}(t,f)=X(t,f).\breve{Y}(t,f)\\ J_{x,x}(t,f)=|X^2(t,f)|,J_{y,y}(t,f)=|Y^2(t,f)|\end{gathered} \tag{7}$$

where $X(t,f)$ and $Y(t,f)$ are the TF transform coefficients of the signal x and y respectively and $\breve{Y}(t,f)$ is the complex conjugate of $Y(t,f)$.

To measure time-frequency coherence between two signals based on synchosqueezing transform can be performed using the following algorithm:

i) Select two signals or signals from two channels
ii) Apply the SST on individual signal/channel to obtain the SST coefficients

iii) Compute cross spectrum and auto spectrum based on the SST coefficients

iv) Finally compute time-frequency coherence using the cross and auto spectrum using Eq. (6).

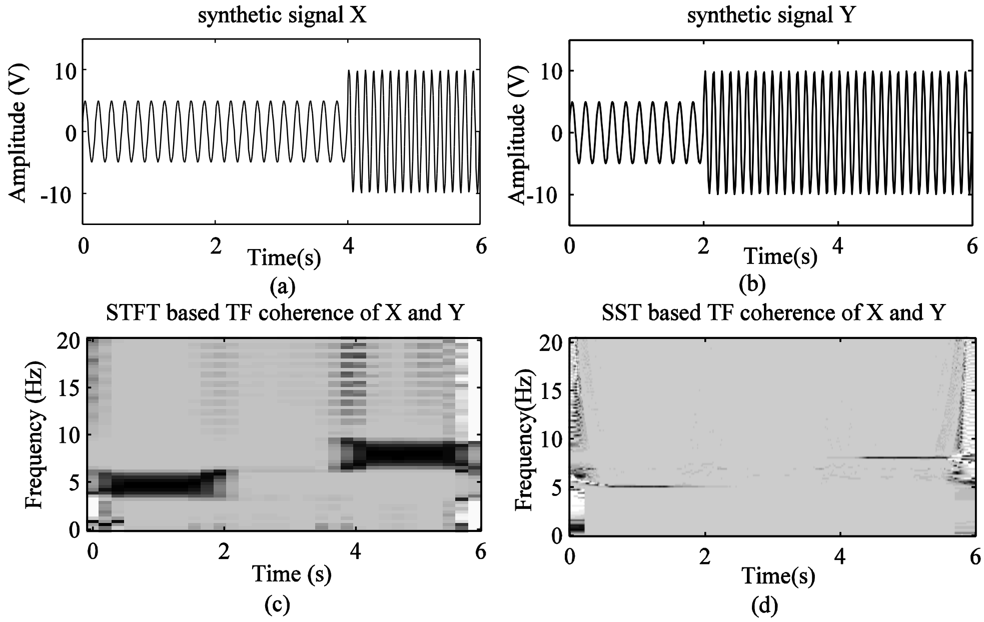

Figure 2: TF coherence analysis (a) synthetic signal *X*, (b) synthetic signal *Y*, (c) STFT based TF coherence between and (d) SST based TF coherence between signal *X* and *Y*

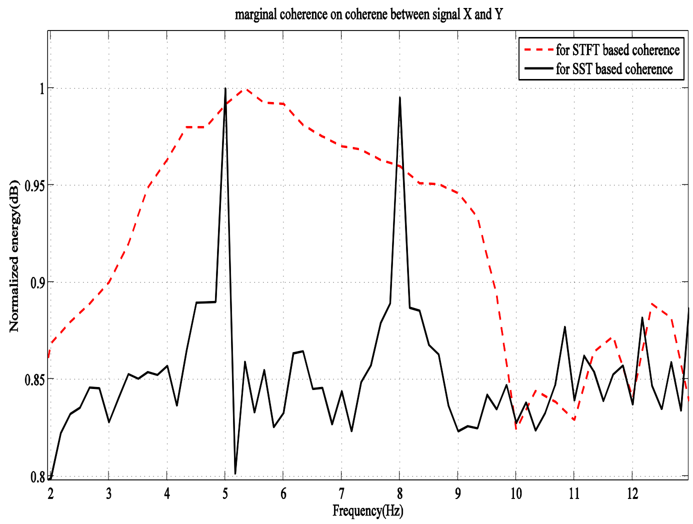

Figure 3: Marginal coherences of STFT based coherence (dashed red line) and of SST based coherence (solid black line) between the synthetic signal X and Y.

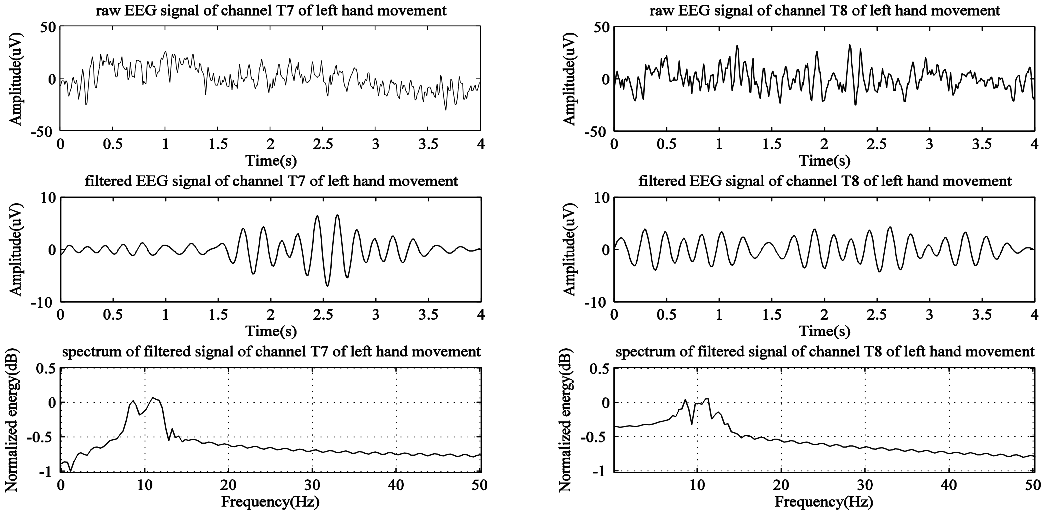

Figure 4: First row is the raw EEG signal of channel T7 and T8 of left hand movement, second row is the filtered EEG signal and the third row is the spectrum of the filtered component.

IV. EXPERIMENTAL RESULTS

The TF coherence between two EEG channels is estimated in this work based on the STFT as well as SST and compared. The performance of the proposed SST based time-frequency coherence is evaluated on both synthetic signals and real data. The experimental results show that the SST based TF coherence illustrates enhanced resolution than STFT. Two synthetic signals X=[sin($2\pi f_1 t$),sin($2\pi f_2 t$)],Y=[sin($2\pi f_1 t$),sin($2\pi f_2 t$)] and their TF coherence are shown in Fig 2, where f_1=5Hz and f_2=8Hz. There are different time alignment of two sinusoids to generate X and Y. It is observed that the proposed method is more efficient than STFT for localization of frequency components with higher resolution in coherence domain. The SST based method represents a sharper coherence frequency definition at 5Hz and 8Hz than that of STFT. In Fig 2 (c), the coherence between signals X and Y (5 Hz and 8 Hz frequency) is overlapped each other, whereas, in Fig 2 (d), the coherence between the same pair of signals is well separated. The phenomenon is clearly illustrated in Fig 3 which represents the marginal coherences of two methods. The marginal coherence is defined as $\tilde{C}_{x,y}(f) = \sum_{t=1}^{T} \left| C_{x,y}(t,f) \right|^2$, for f=1,2,…,F. With STFT, the energies in coherence domain of closer frequencies are overlapped and that with SST sharply represents the contribution of individual frequencies. It is observed that the STFT based time-frequency coherence exhibits poor resolution than SST based method.

The real electroencephalography (EEG) data collected from the Brain Computer Interface (BCI) Competition IV dataset are also used to evaluate the performance of the proposed method. The data is recorded from healthy subjects. In the whole session motor imagery is performed without feedback. For each subject two classes of motor imagery are selected from left hand, right hand, and foot. The calibration data ds1a of the BCI competition IV are continuous signals of left hand and foot movement. The data contains 59 EEG channels, total 200 trials with four second each. The sampling rate of the data is 100 Hz. As a pre-processing, the data offset has been removed from the EEG signals. Then the signal is passed through a band pass filter of the range between 8Hz and 12Hz to obtain alpha frequency band. Two channels T7 and T8 are chosen to measure the coherence in this experiment.

The raw EEG signal (first row), the filtered alpha component (second row) and the spectrum of the alpha component (third row) of channels T7 and T8 of left hand movement are shown in Fig 4. The STFT based time-frequency coherence between channel T7 and T8 for left hand movement motor imagery is shown in Fig 5(a). The time-frequency coherence based on SST between channel T7 and T8 of left hand movement is represented in Fig 5(b). The energy corresponding to the marginal coherence is illustrated in Fig 6 for left hand data. The marginal coherence based on STFT represents poor localization of frequency components, whereas, SST based method illustrates sharp localization of each component within very narrow band of frequencies. In Fig 6, the frequencies of 8.5Hz and 11Hz are well separated with SST based marginal coherence but it is unable to separate those frequencies in STFT based approach. Hence the SST based time-frequency coherence is superior to that of using STFT. The underlying reason is that, the energy in STFT spreads over a wide range of frequency due to the use of window function with overlapping which introduces cross-spectral energy.

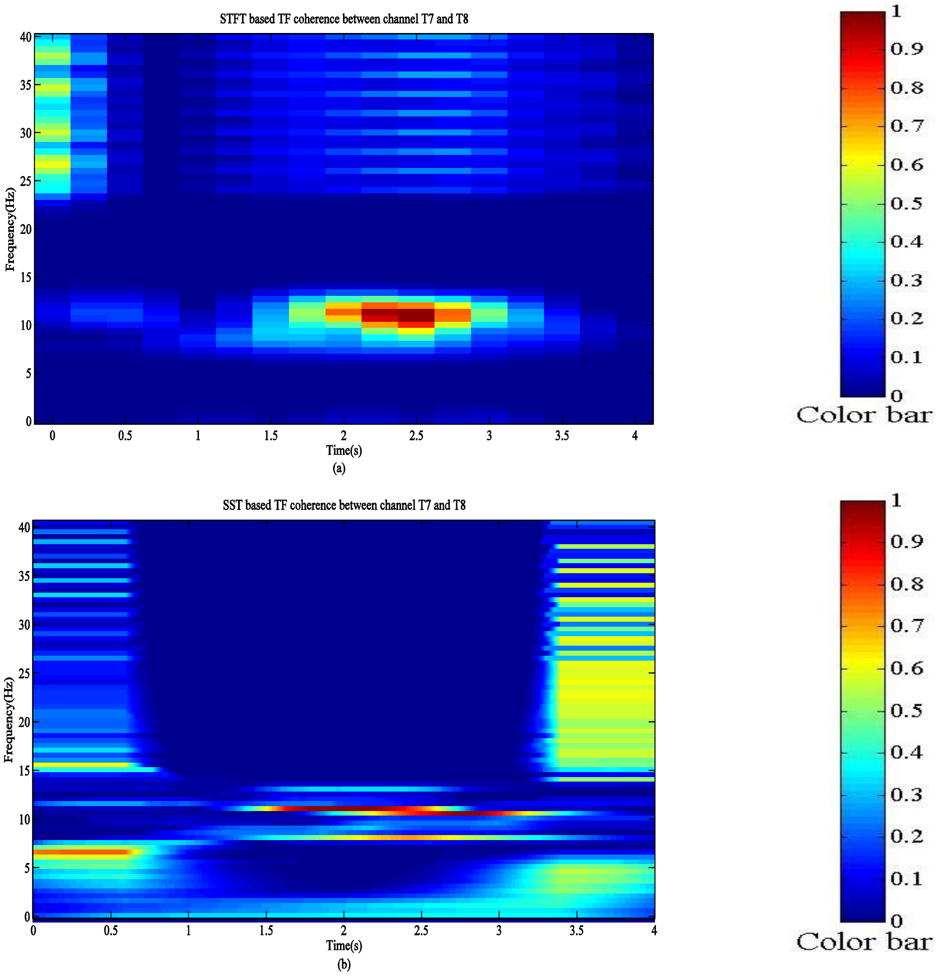

Figure 5: (a) STFT based TF coherence and (b) SST based TF coherence between channel T7 and T8 of left hand movement data.

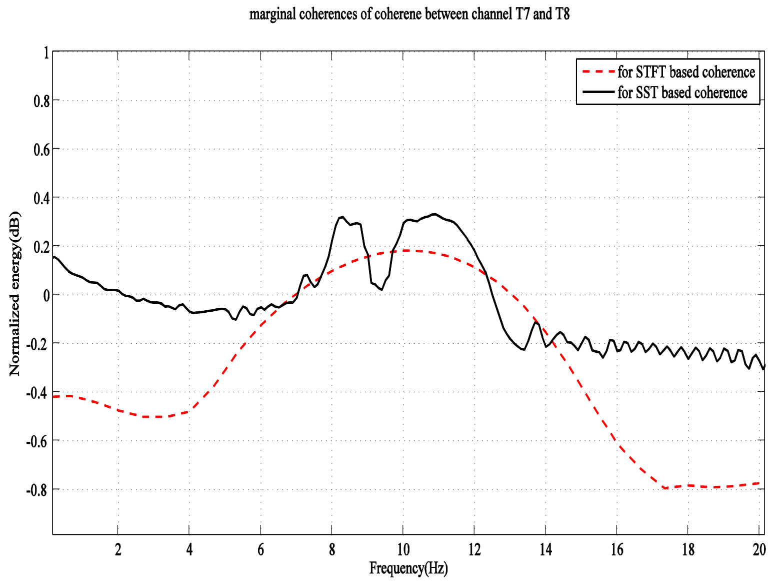

Figure 6: Marginal coherences of STFT based coherence (dashed red line) and of SST based coherence (solid black line between channel T7 and T8 of left hand movement data.

V. CONCLUSION

A novel method to analysis the time-frequency (TF) coherence between a pair of signals is introduced in this paper. The cross and auto spectrums are calculated for the given signals in time frequency domain. The TF coherence is estimated with the spectra for synthetic signals with STFT and SST based time-frequency representation. The proposed SST based coherence estimation method is applied to real EEG signals of different motor imagery. The performance is compared with STFT based measure of TF coherency. It is observed that SST based method is more efficient than STFT for localization of frequency components with higher resolution in coherence domain.

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