

Fusion of Compressed Sensing Algorithms for ECG Signals

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ABSTRACT

Wireless Body Area Networks (WBANs) is a promising technology providing pervasive healthcare monitoring systems. Rapid improvement in the area of low-power operated integrated circuits, physiological sensors have facilitated wireless sensor networks as a new generation of wireless communication technology. A BAN is an integrated area which enables continuous low-cost health monitoring by providing real time updates to the medical personnel with increased mobility and quality of the healthcare systems.

Cardiac diseases are often in present day and thus continuous cardiac monitoring (electrocardiogram) and early detection is mandatory. This continuous monitoring of ECG produces huge amount of data which requires enormous storage memory. Hence sensed data from WBAN nodes requires compression before transmission. ECG signal is compressed using Compressed Sensing algorithms. Number of parameters can be observed from these algorithms. As none of these algorithms outperforms other algorithms with respect to all parameters, there is a need for Fusion technique which fuses the final estimates of the chosen algorithms to produce a better result. All these algorithms are implemented and tested using standard ECG records taken from the MIT-BIH arrhythmia database. The performance parameters are compared with respect to individual algorithms and fusion algorithms. The dissertation is finally concluded by upholding that the fusion of algorithms performs better reconstruction as compared to individual algorithms.

Keywords :— *Electrocardiography; wireless body area networks (WBAN); Compressed sensing (CS).*

I. INTRODUCTION

In the present era, globalization has influenced the medical field. Many diagnostic centers are globally implementing wireless media for the exchanging of bio-medical information for efficient and early analysis of the diseases as well as to improve the diagnostic results. The biomedical signals are transmitted using communication channels and are emerging as an important issue in health monitoring applications. The elder population in developed countries and the rising costs of health care has triggered the introduction of the novel technology based enhancements in health care practices. The main health aspect in present day world is Cardio Vascular Diseases (CVD). The major cause is diabetes because of which more than 246 million people are suffering, and by 2025 this number is expected to rise to 380 million [1].

Electrocardiogram (ECG) is used for diagnosing Cardio Vascular Disease of a patient. In an ambulatory system, the ECG data is of very large size, because long time is taken to collect enough information about the patient. Cardiac patients can be monitored continuously with the assistance of wearable ECG sensors. ECG is the simple representation of the electrical activity which occurs in the heart muscle as it changes with time. For easy analysis of ECG it is usually printed on paper. High security and authentication is required during the communication of these medical data through wireless media. As the continuous monitoring of ECG signals is very necessary, with the sampling rate of 360 Hz, 11 bits/sample data resolution, a 24-hour record requires about 43

Mbytes per channel [2]. Thus, there is a need for some technique which reduces the memory required for storage. Compressed Sensing is one of the techniques to achieve this goal. Compressed Sensing is a signal processing technique using which one can acquire and recover a signal with less number of samples as the signal is sampled at a rate less than Nyquist sampling rate. As ECG signal is a compressible signal, compressed sensing technique can be employed for ECG signals. Compressed sensing comprises a number of algorithms using which reconstruction of the signal can be performed. All the parameters are not explained with a single CS reconstruction algorithm. Thus it is not possible to choose a better sparse reconstruction algorithm from set of available algorithms. To overcome this problem, fusion of estimates of the viable algorithms is one of the solutions [3].

In today's digital world, real-time health monitoring is becoming an important issue in the field of medical research. Body signals such as ECG, EMG, EEG etc., are produced in human body. Cardio vascular diseases are becoming very common in present day and the continuous monitoring of ECG signals is very necessary to understand the health condition of a patient. This continuous monitoring produces large amount of data and thus an efficient method is required to reduce the size of this large data. Compressed Sensing (CS) is one of the techniques used to compress the data size. Compressed Sensing technique comprises numerous algorithms for compression and reconstruction of a signal. Parameters such as signal dimension, sparsity level, noise power, number of measurements, the occurrence of the non-zero elements of the sparse signal affects the performance of these algorithms. Thus, an efficient technique is necessary to estimate all the

signal parameters and also to reconstruct the signal with a better quality.

The performance of the Compressed Sensing algorithms are highly influenced by the signal parameters such as dimension, sparsity level, number of measurements, and the alignment of non-zero elements of the sparse signal. Based on the literature survey, it is understood that none of the existing CS algorithms can estimate all these parameters of a signal accurately. Hence, to get an improved and accurate statistics Fusion of more than one CS algorithms is possible.

For efficient reconstruction and to estimate signal parameters, Compressed Sensing algorithms are executed independently and the estimate of these algorithms are fused to obtain a final estimate. There are numerous algorithms proposed under Compressed Sensing. In this dissertation, Orthogonal Matching Pursuit (OMP) algorithm, Subspace Pursuit (SP) algorithm and Compressive Sampling Matching Pursuit (CoSaMP) algorithm are studied and implemented.

II. COMPRESSED SENSING

The current healthcare monitoring systems are to be enhanced to enable patients with their health conditions, and also to the medical personnel so that they can grant efficient information with increased mobility. Healthcare monitoring system is a continuous long process. Thus the quality of data quickly increases. To achieve efficient storage and to reduce transmission time and the power consumption, sampling and compression are performed simultaneously in Wireless Healthcare Systems (WHSs). WHSs transmits vital information gathered from the human body to the medical personnel, by restraining location of the patient and time, and also by increasing mobility and offering better quality.

Compressed Sensing (CS) theory is used in healthcare systems operating at low power. CS theory implicates simultaneous sampling and compressing of a signal. Basic idea of CS theory is, when a signal is sparse, small number of coefficients is sufficient to reconstruct the signal. Thus, signal is obtained directly in compressed representation form.

Compressive Sensing is a useful concept while dealing with redundant, limited data. “Compressive sensing (CS) is a new approach for the acquiring and recovering a sparse signals which results in sampling rates significantly below the classical Nyquist rate”[9]. Acquisition and compression are the important stages in processing data in which the compression is carried either using a dedicated algorithm explicitly or implicitly as a part of any inference methodology. Compression stage involves eliminating redundancies and insignificant part of a signal within the data for producing a concise representation. Thus in compression stage required

part of data is retained and unwanted data part is excluded. In data acquisition stage, data sufficient for processing is collected. Hence Compressive Sensing is used in concepts like reconstructing the compressed representations of mathematical objects by using limited data, which is very less than the objects ambient dimension (i.e., the dimension of the object when it is uncompressed).

“Compressive sensing theory states that a small number of random linear measurements contain enough information to recover the original signal”[9]. Figure 1 shows block diagram of Compressive sensing.

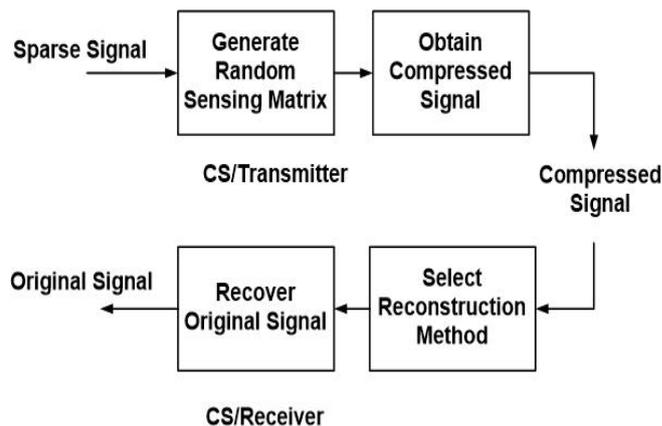


Figure 1: Transmitter and Receiver of Compressive sensing concept

Shannon sampling theorem has two major drawbacks:

1. They produce very large number of samples for large bandwidth applications
2. They produce huge amount of redundant digital samples for low bandwidth applications.

Thus, fundamental aim of compressive sampling theory is to reduce the number of measurements needed to acquire a signal completely. CS theory depicts that a sparse signal in terms of small number of non-zero entries or coefficients, can be represented accurately as a random linear combinations of some projections of such signals in data independent random vectors. A general linear measurement technique computes the inner product between the original signal and a random sensing matrix. An important concept in CS theory is that the element of sensing matrix ϕ is linear combination of the signal measurements. A compressible or a sparse signal in R to the power of N can be expressed in terms of a suitable sparse basis is given by:

$$c = \sum_{i=1}^N D_i \varphi_i \tag{1}$$

The compressible signal has many K non-zero coefficients and (N - K) zero coefficients with $K \ll N$. If many natural signals are compressible then sparsity is determined. If a basis exists whose representation shown in Equation 3.1 depicts that it has few large coefficients and many small coefficients and they are well approximated by K-sparse representations. Thus the compressible signal C is a linear combination of only K basis vectors, where $K \ll N$ and is approximated using K-sparse representation. Compressible signals such as data networks, data of WSNs, data of digital images, data of biomedical systems, and data of A/D invertors have K nonzero coefficients, but their location is unknown.

As compared to a wavelet based compression algorithm, which is inefficient to find the location of nonzero coefficients, one must compute N number of samples which is a large number even if the desired K is small and the encoder must compute all of the N transform coefficients S_i even though it discards ($N < K$) of them This produces the overhead problem in encoding the locations of the large coefficients. Thus CS represents the large samples in terms of linear random measurements in compressed signal and also it offers stable measurements matrices with M independent and identically distributed (i.i.d) elements of the compressed signals such as $K < M < N$. From these M random linear measurements CS guarantees the recovery of signal from the compressed signal. In digital-CS theory, any compressible or sparse signal C in R^n can be expressed as

$$c = \sum_{i=1}^N D_i \varphi_i \tag{1}$$

The compressed signal D is found as:

$$[D]_{M \times 1} = [\Phi]_{M \times N} [C]_{N \times 1} \tag{2}$$

$$[D]_{M \times 1} = [\Phi]_{M \times N} [\varphi]_{N \times N} [D]_{N \times 1} [\Gamma]_{M \times N} [D]_{N \times 1} \tag{3}$$

They are incoherent with the basis and they have the Restricted Isometry Property (RIP) with accurate level for detection probability of the compressed signals at the receiver part that is suitable for the recovery the original signal from compressed signal. CS scenario has two important steps.

- a. A measurement matrix $[\Phi]_{M \times N}$ should make sure that the important features of an compressible signal is not damaged by the dimensionality reduction from C belongs to R^N to D belongs to R^M .
- b. The CS theory offers a reconstruction algorithm under some conditions with enough accuracy to recover signal C from the compressed signal.

Therefore, CS theory mainly focus on M random linear measurements instead of N samples such that $M \ll N$. The CS theory also offers a reconstruction algorithm to recover signal C from the compressed signal D only with only M random linear measurements. The number of random linear measurements and non-zero coefficients must satisfy the condition given in 4:

$$M < K/R \log(N) \tag{4}$$

Where R is a constant ($R=1.5$), and M, N, and K are the number of random measurements, the total of coefficients, and the number of non-zero coefficients respectively. The CS theory consists of the following steps:

- a. Design of a random measurement matrix.
- b. To develop a suitable reconstruction algorithm using which original signal can be recovered from the compressed signal at the receiver.
- c. Test the number of measurements M to make sure that the reduction of samples from N to M is affected the recovery of the original signal.

The measurement matrix or the sensing matrix can constructed using random matrices with independent identically distributed (i.i.d) entries. It is formed either by Gaussian distribution or by Bernoulli distribution.

Gaussian distribution: Here the entries of the sensing matrix are independently Sampled from the normal distribution with zero mean and variance $1/N$.

$$\phi \sim N\left(\frac{0,1}{N}\right) \tag{5}$$

Bernoulli distribution: In these random matrices the entries are independent realizations of Bernoulli random variables

$$\phi = \begin{cases} +\frac{1}{\sqrt{N}} \\ -\frac{1}{\sqrt{N}} \end{cases} \text{ with probability } \frac{1}{2} \tag{6}$$

Compressive sensing (CS) is a new approach for acquiring and recovering a sparse signal at sampling rates less than classical Nyquist rate. If a signal is sampled at a rate less than the Nyquist sampling rate, then it is referred as a sparse signal. In a compressible signal, the coefficients of a vector consists of few large coefficients at certain basis and remaining coefficients with less value. These small coefficients are made zero, so that remaining large coefficients can represent the original signal with unnoticeable loss. If the number of non zero entries are k then we call this modified vector of coefficients with k non-zero entries as k -sparse signal.

We say that a signal x is k -sparse when $\|x\|_0 < K$. All real signals are quite often compressible signals, so that their entries tend to zero when sorted by magnitude. Hence, compressible signals are approximated by sparse signals. Thus, dimensionality reduction can be achieved using a sparse signal.

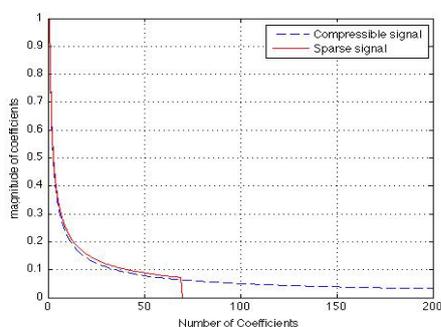


Figure 2: Comparison of Sparse signal and a Compressible signal

In compressive sensing, a sample (co-ordinates of a randomly projected signal), is a linear function applied to a signal. Multiple samples are acquired by multiplying measurement matrix with the input signal. If we take M samples, or measurements, of a signal in \mathbb{R}^N , then the measurement matrix has dimensions $M \times N$. The minimum number of measurements required to obtain a k -sparse signal is $M < 2k$ which obeys that the matrix should not map two different k -sparse signals to same set of samples. Therefore, each combination of $2k$ columns of the measurement matrix must be nonsingular. Consider a standard Compressed Sensing setup [3] for obtaining a signal x belongs to \mathbb{R}^N which is k -sparse using linear measurement as:

$$y = \phi x + w \tag{7}$$

Where ϕ belongs to $M \times N$ represents a measurement matrix, y belongs to $M \times 1$ represents a

measurement vector and w belongs to $M \times 1$ denotes additive noise. As we know that a K -sparse signal consists of at most K non-zero entries such as $\|x\|_0 < K$.

In CS-setup reconstruction of sparse signal comprises:

1. Estimating the sparsity level K assuming $K < M < N$
2. Identifying non-zero elements indices (it is known as support-set)
3. Estimation of magnitude of non-zero elements.

III. COMPRESSED SENSING ALGORITHMS

In Fusion of algorithms, several CS reconstruction algorithms are executed independently and in parallel. In this dissertation, three CS reconstruction algorithms namely, Orthogonal Matching Pursuit algorithm (OMP), Subspace Pursuit algorithm (SP) and Compressive Sampling Matching Pursuit algorithm (CoSaMP) are implemented.

Orthogonal Matching Pursuit:

Orthogonal Matching Pursuit (OMP) is an iterative greedy algorithm which selects the column of ϕ in each step, which is highly correlated with the remaining samples of the signal. Then this sample is subtracted from measurement vector and the residual is updated. OMP has to determine which column has to be participate in the measurement vector, a partially known support is necessary to improve the recovery of the signal. This partially known support leads to a priori information of few columns which are to be selected. This algorithm is used for recovering a high-dimensional sparse signal based on less number of noisy linear measurements, which is a basic problem in signal in signal processing.

Algorithmic steps

Consider the following model:

$$x = \phi y$$

Inputs to the algorithm:

- Measurement dictionary ϕ
- Measurement signal x
- Sparsity level k

Initialization:

- Index set $\Lambda = \emptyset$ (estimated support set)
- Residual signal $r = x$
- Step 1: Identification

$$i = \max_{1 \leq j < n} |\phi_j^T r|$$
- Step 2: Sparse signal Reconstruction

$$\Lambda = \Lambda \cup i$$

$$\tilde{y} = \Phi_{\Lambda}' x$$

- Step 3: Residual Update

$$r = r - \Phi_{\Lambda} \tilde{y}$$
- Repeat step 1 to 3 for k iterations.

Output:

- Reconstructed sparse signal \hat{y} .

Φ_{Λ} represents the matrix formed by columns of Φ with index in the set Λ in each iteration OMP selects the atom of dictionary Φ which is highly correlated with current residual r. In Step 2, the index of this atom is added into the set of selected atoms Λ . In OMP algorithm, once one atom is selected and its index is added to the index set Λ , residual signal is updated.

Subspace Pursuit:

The Subspace Pursuit algorithm is an iterative algorithm used to find K columns that span the subspace correctly. SP algorithm tests the subsets of K columns in a group. To achieve this purpose an initially chosen estimate for the subspace is refined at each stage. This algorithm has two important characteristics:

1. Computational complexity is less, as compared to orthogonal matching pursuit techniques when an highly sparse signal is considered.
2. Reconstruction accuracy is very high.

Algorithmic steps:

Consider the following model:

$$x = \Phi y$$

Inputs to the algorithm:

- Measurement dictionary Φ
- Measurement signal x
- Sparsity level k
- Index set $\Lambda = \Phi$ (estimated support set)
- Residual signal r = x
- Step 1: Using identification vector find an initial support-set

$$i = \underset{1 < j < n}{\text{sort}} |\varphi_j^T r|$$
 (Descending order)
- Step 2: Residual Update

$$\tilde{y} = \Phi_{\Lambda}' x$$

$$r = r - \Phi_{\Lambda} \tilde{y}$$

$$\text{Res_n1} = \text{norm}(r, 2)$$

- Set counter to M iterations.
- Step 3: Assign previous estimated support set

$$\Lambda_p = \Lambda$$
- Step 4: Identification vector 1

$$i = \max_{1 < j < n} |\varphi_j^T r|$$
 (Descending order)
- Step 5: Intermediate estimated support set

$$\Lambda_m = \Lambda \cup i_1(1:k)$$
 Where

$$k < |\Lambda| < 2k$$
- Choose best K using the intermediate estimate of signal \bar{y} .

$$\bar{y}(\Lambda_m) = \Phi_{\Lambda} \bar{y}$$
 Where

$$\tilde{y} = \Phi_{\Lambda}' x$$
- Identification vector 2

$$i_2 = \text{sort} |\bar{y}|$$
 (Descending order)
- Step 6: Update estimated support set

$$\Lambda = i_2'$$
- Step 7: Update residue

$$r = r - \Phi_{\Lambda} \bar{y}$$
- Step 8: Check for halting condition

$$\text{Res_n2} = \text{res_n1}$$

$$\text{Res_n1} = \text{norm}(r, 2)$$

$$\text{If } (\text{res_n2} _ \text{res_n1}) \text{ then } \Lambda = \Lambda_p$$

Output:

- Obtain final estimate

$$\hat{y}(\Lambda) = \tilde{y}$$

Compressive Sampling Matching Pursuit:

Developing sampling technologies for the purpose of dimension reduction is the main idea behind compressive sampling (CoSa). The fact that by using a suitable random projection a compressible signal retains much information of a signal forms the mathematical foundation for this approach. In this randomly projected signal the coordinates are referred as samples. CoSaMP is similar to OMP in which, initialization needs to be bit modified and the residual is obtained by subtracting the contribution of the first estimate. But this algorithm offers high computational cost and storage.

Algorithmic steps:

Consider the following model:

$$x = \Phi y$$

Inputs to the algorithm:

- Measurement dictionary Φ
- Measurement signal x

- Sparsity level k
- Index set $\Lambda = \Phi$ (estimated support set)
- Residual signal $r = x$
- $\hat{y} = \text{zeros}(N,1)$
- Res_n1 = norm(r; 2)
Set counter to M iterations.
- Step 1: Assign previous estimated support set
 $\Lambda_p = \Lambda$
 $\hat{y}_p = \hat{y}$
- Step 2: Find large component using identification vector 1
 $i = \text{sort}_{1 < j < n} |\varphi_j^T r|$ (Descending order)
- Step 3: Merge support set
- $\Lambda_m = \Lambda \cup i_1$ (1:2K)
Where $k < |\Lambda| < 3K$
- Step 4: Choose best K using the intermediate estimate of signal \bar{y} .
 $\bar{y}(\Lambda_m) = \Phi_{\Lambda} \bar{y}$ Where
 $\bar{y} = \Phi_{\Lambda}^T x$
- Step 5: Identification vector 2
 $i_2 = \text{sort}|\bar{y}|$ (Descending order)
- Step 6: Update estimated support set
 $\Lambda = i_2'$
- Step 7: Obtain Estimate of the signal
 $(\Lambda) = \bar{y}(\Lambda)$
- Update current samples (CoSaMP methodology)
 $r = x - \Phi$
- Step 8: Check for halting condition
Res_n2 = res_n1
Res_n1 = norm(r, 2)
if (res_n2 < res_n1) then
 $\Lambda_p = \Lambda$

Output:

- Final reconstructed sparse output
 $\hat{y} = \hat{y}_p$

Fusion based Compressed Sensing algorithms framework

- Step 1: Obtain Union Set
 $\text{unionset}(U_s) = \text{omp}U_{SP}U_{CoSaMP}$
- Check condition
 $\text{unionset} < M$
- Find intermediate set using union set
 $\bar{y}_U(U_s) = \Phi_{\Lambda}^T x$

- Find index set
 $i = \text{sort}|\bar{y}_U|$
- Find the estimated support set
 $\Lambda = i(1:k)$
- Find the final estimate of the signal
 $\hat{y}(\Lambda) = \Phi_{\Lambda}^T x$

IV. RESULTS

Results of OMP:

The algorithm is implemented using the ECG records taken from MIT-BIH database. The results of OMP algorithm obtained for record 103 are listed. Figure 4.1 is the input ECG signal. The first 500 samples of the signal are represented as waveform.

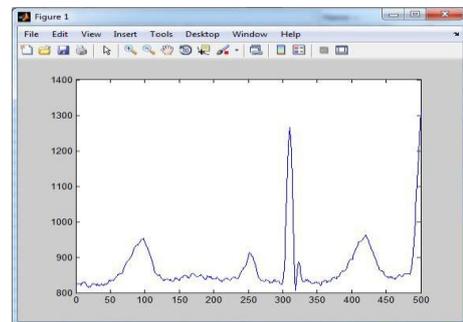


Figure 3: ECG input signal of record 103

The identification vector indicating the absolute maximum value of the inner product of the signal with the sensing dictionary is shown in figure 4. The ECG signal is reconstructed using OMP algorithm with less loss in the signal information.

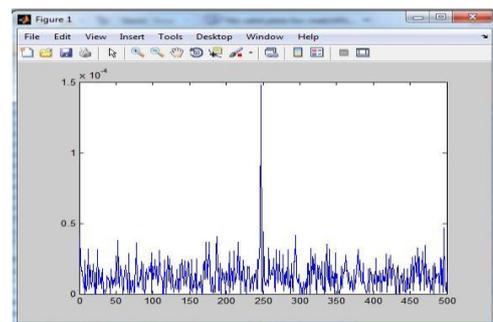


Figure 4: Step1: Identification

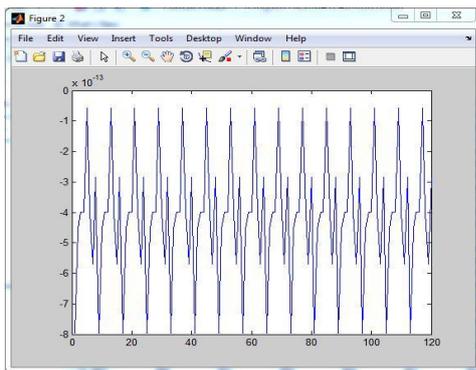


Figure 4: Step2: Residual update

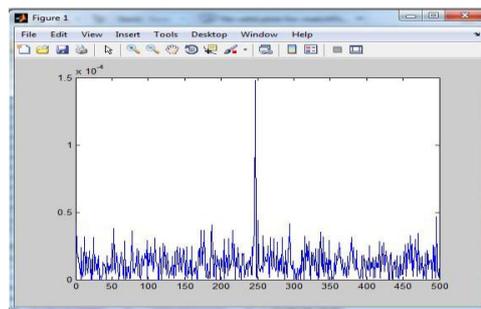


Figure 7: Step1: Identification

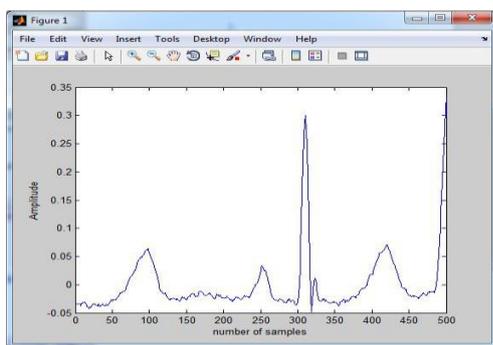


Figure 5: Step3: Reconstructed ECG signal

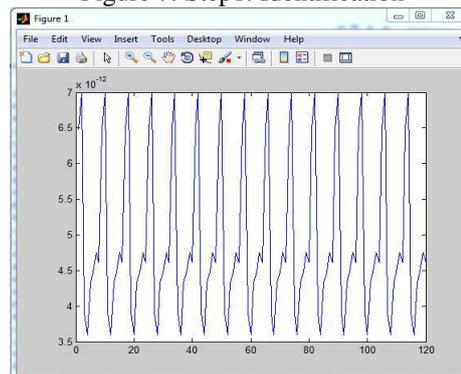


Figure 8: Step2: Residual update

Results of SP:

The record 103 taken from the database is used for testing SP algorithm. The first 500 samples is taken for testing and is shown in figure 6. The identification vector indicating the maximum absolute value given in step 1 is plotted in figure 7. The final reconstructed signal obtained by reducing the samples is shown in figure 9.

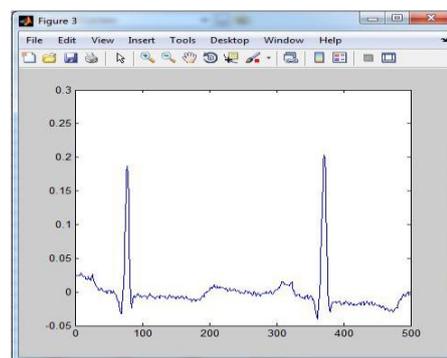


Figure 9: Step3: Reconstructed sparse signal

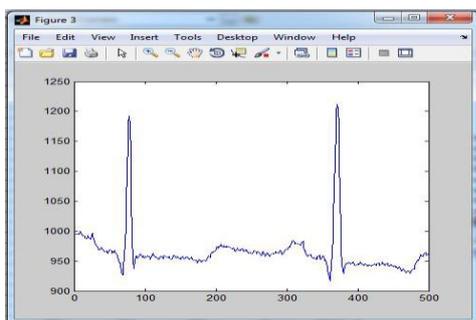


Figure 6: Input ECG signal of record 103

Results of CoSaMP:

The first 500 samples of record 103 is chosen as input signal for testing CoSaMP algorithm. Shown as in figure 10. The identification vector mentioned in step 2 indicating the maximum absolute value of the signal is shown in figure 11. The final reconstructed sparse signal is shown in figure 13.

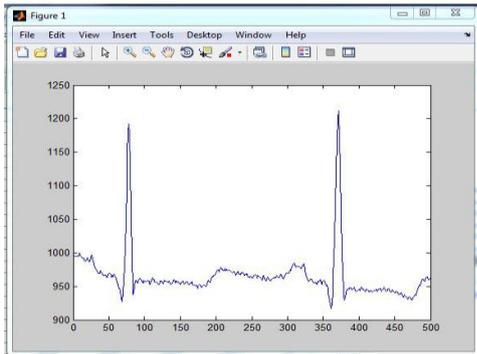


Figure 10: ECG input signal

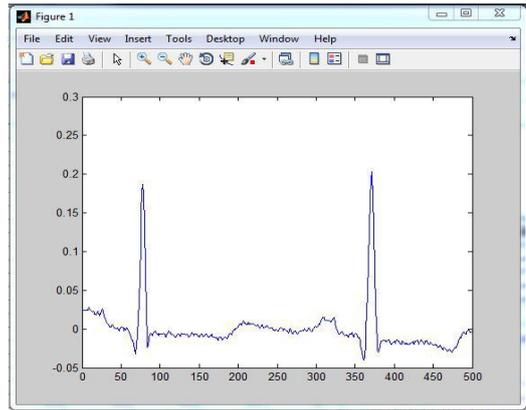


Figure 13: Step3: Reconstructed sparse signal

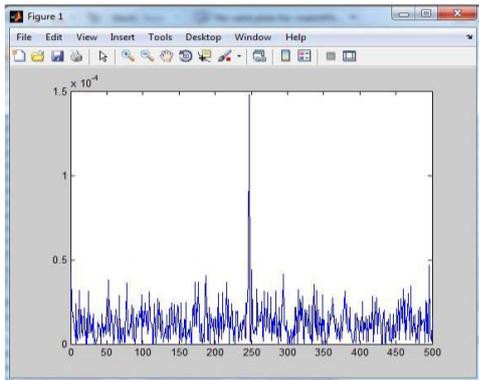


Figure 11: Step1: Identification

The performance parameter SRER obtained for OMP, SP, and Fusion of OMP and SP is shown in figure 14. From the plot we can observe that the SRER at threshold 0.2 for fusion algorithm is exceeded by 12dB indicating that the fusion algorithm is better reconstructing the signal.

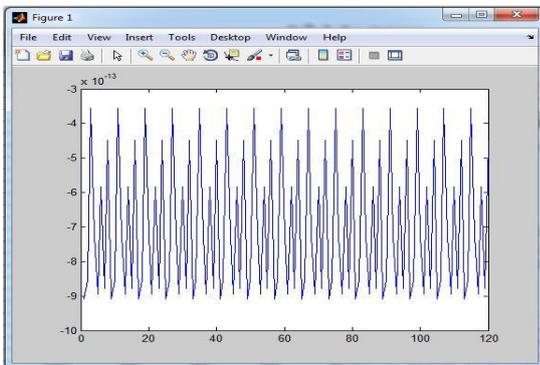


Figure 12: Step2: Residual update

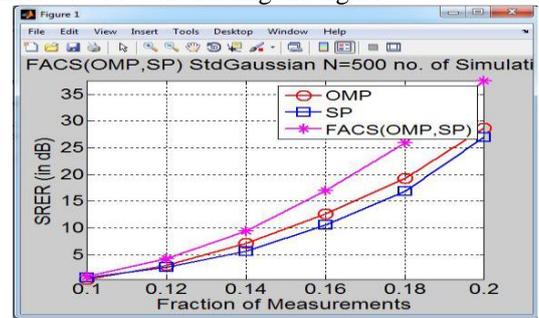


Figure 14: SRER value for fusion of OMP and SP algorithms

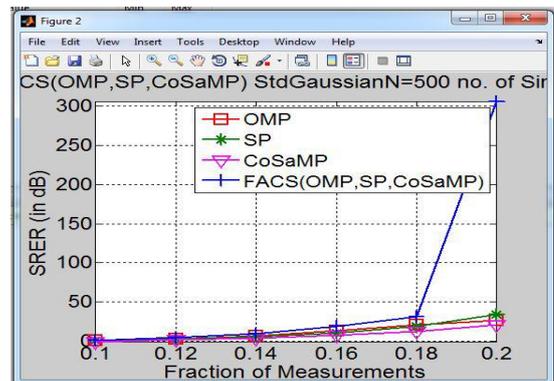


Figure 15: SRER value for fusion of OMP, SP and CoSaMP algorithms

V. CONCLUSION

In this dissertation, Compressed Sensing algorithms are used to test the Compression of ECG signals. The sparse nature of the ECG signal is studied. The algorithms are implemented in MATLAB 2011 software. The sparsification of the real ECG signal has great influence on the compressibility of the signal. It is observed that the fusion of outputs from several CS algorithms provides better reconstruction performance along with increasing robustness. These algorithms are tested using several records taken from standard MIT-BIH arrhythmia database. The performance parameters like SRER, CR, PRD, SNR and computational time are measured.

- The SRER value has a significant rise in the fusion algorithms as compared to individual algorithms.
- The result analysis of the simulation illustrates considerable PRD, reconstruction quality of the signal after applying CS algorithms is between 4-9% which is good as per [2] but needs to be improved for real-time deployment on WBAN nodes.
- The SNR value is acceptable for preserving the salient features of the ECG biomedical.
- However the speed of execution is on an average around 12 seconds which is not suitable for Real-time alert generation and delivery of information to care taker or the doctor. Hence needs to be reduced to meet the real-time constraints.

Eventually, we can conclude that there is strong need for trade-off between PRD, SNR, SRER and CR parameters which needs to be optimized. There exists a large class of CS algorithms which can also be used for implementation. The execution time needs to be reduced using some technique to meet the real-time constraint. By using sophisticated

strategies, the performance of the fusion based algorithms can be enhanced.

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