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On The Greatest Splitting Topology

L.Sowmyalatha [1], K.Thirunanasambandam [2]

Associate Professor
Department Of Mathamatics
DKM College for Women [1]
Muthurangam Government Arts College [2]
Vellore – 632002
Tamil Nadu - India

ABSTRACT

It is well known that (see, for example, [H. Render, Nonstandard topology on function spaces with applications to hyperspaces, Trans. Amer. Math. Soc. 336 (1) (1993) 101–119; M. Escardo, J. Lawson, A. Simpson, Comparing cartesian closed categories of (core) compactly generated spaces, Topology Appl. 143 (2004) 105–145; D.N. Georgiou, S.D. Iliadis, F. Mynard, in: Elliott Pearl (Ed.), Function Space Topologies, Open Problems in Topology, vol. 2, Elsevier, 2007, pp. 15–22]) the intersection of all admissible topologies on the set $C(Y, Z)$ of all continuous maps of an arbitrary space Y into an arbitrary space Z, is always the greatest splitting topology. However, this intersection maybe not admissible. In the case, where Y is a locally compact Hausdorff space the compact-open topology on the set $C(Y, Z)$ is splitting and admissible (see [R.H. Fox, On topologies for function spaces, Bull. Amer. Math. Soc. 51 (1945) 429–432; R. Arens, A topology for spaces of transformations, Ann. of Math. 47 (1946) 480–495; R. Arens, J. Dugundji, Topologies for function spaces, Pacific J. Math. 1 (1951) 5–31]), which means that the intersection of all admissible topologies on $C(Y, Z)$ is admissible. In [R. Arens, J. Dugundji, Topologies for function spaces, Pacific J. Math. 1 (1951) 5–31] an example of a non-locally compact Hausdorff space Y is given having the same property for the case, where $Z =[0,1]$, that is on the set $C(Y,[0,1])$ the compact-open topology is splitting and admissible. This space Y is the set [0,1] with a topology τ, whose semi-regular reduction coincides with the usual topology on [0,1]. Also, in [R. Arens, J. Dugundji, Topologies for function spaces, Pacific J. Math. 1 (1951) 5–31, Theorem 5.3] another example of a non-locally compact space Y is given such that the compact-open topology on the set $C(Y,[0,1])$ is distinct from the greatest splitting topology.

In this paper first we construct non-locally compact Hausdorff spaces Y such that the intersection of all admissible topologies on the set $C(Y, Z)$, where Z is an arbitrary regular space, is admissible. Furthermore, for a Hausdorff splitting topology t on $C(Y, Z)$ we find sufficient conditions in order that t to be distinct from the greatest splitting topology. Using this result, we construct some concrete non-locally compact spaces Y such that the compact-open topology on $C(Y, Z)$, where Z is a Hausdorff space, is distinct from the greatest splitting topology. Finally, we give some open problems.

Keywords:- Splitting topology,Admissible topology,Greatest splitting topology,Semiregularity

I. PRELIMINARIES

Let Y and Z be two spaces. If t is a topology on the set $C(Y , Z)$ of all continuous maps of Y into Z, then the corresponding space is denoted by $C_t(Y , Z)$. A topology t on $C(Y , Z)$ is called *splitting* if for every space X, the continuity of a map $g : X \times Y \to Z$ implies that of the map $g : X \to C_t(Y , Z)$ defined by relation $g(x)(y) = g(x, y:)$ for every→ $x \in X$ and $y \in Y$. A topology t on $C(Y , Z)$ is called *admissible* if for every space X, the continuity of a map $f\, X\, C_t(Y , Z)$ implies that of the map $f : X \times Y \to Z$ defined by relation{ ∈} for every $(x, y) \in X \times Y$ (see [1,6,2]).

Let t be a topology on $C(Y, Z)$. If a net $f_\mu, \mu \in M$ of $C(Y, Z)$ converges topologically to an element f of $C(Y, Z)$, then we write $\{ f_\mu, \mu \in M \} \xrightarrow{t} f$. (If M is the set ω of all non-negative integers, then the net is called a sequence.)

A net $\{ f_\mu, \mu \in M \}$ of the set $C(Y, Z)$ *converges continuously* to $f \in C(Y, Z)$ (see [7] and [12]) if and only if for every $y \in Y$ and for every open neighborhood W of $f(y)$ in Z there exists an element $\mu_0 \in M$ and an open neighborhood V of y in Y such that for every $\mu \geqslant \mu_0$, $f_\mu(V) \subseteq W$.

A subset B of a space X is called *bounded* (see, for example, [14]) if every open cover of X contains a finite subcover of B. A space X is called *corecompact* (see, for example, [10]) if for every open neighborhood U of a point $x \in X$ there exists an open neighborhood $V \subseteq U$ of x such that V is bounded in the space U.

In 1972, D. Scott defined a topology on a partially ordered set L which is known as the Scott topology (see, for example, [10]). If L is the set $O(Y)$ of all open sets of the space Y partially ordered by inclusion, then the Scott topology coincides with a topology defined in 1970 by B.J. Day and G.M. Kelly (see [3]): a subset $\mathbb{H}$ of $O(Y)$ is an element of this topology (that is, the Scott topology) if and only if: (α) the conditions $U \in \mathbb{H}$, $V \in O(Y)$, and $U \subseteq V$ imply $V \in \mathbb{H}$, and (β) for every collection of open sets of Y, whose union belongs to $\mathbb{H}$, there are finitely many elements of this collection whose union also belongs to $\mathbb{H}$.

The *Isbell topology* on $C(Y, Z)$, denoted here by t_{Is}, was defined by J.R. Isbell in 1975 (see [11,15,10]): a subbasis for t_{Is} is the family of all sets of the form

$$(\mathbb{H}, U) = \{ f \in C(Y, Z) : f^{-1}(U) \in \mathbb{H} \},$$

where $\mathbb{H}$ is an element of the Scott topology on $O(Y)$ and U is an open subset of Z (see [13]). It is well known that:

(1) A topology t on $C(Y, Z)$ is splitting if and only if each net of elements of $C(Y, Z)$ converging continuously to an element of $C(Y, Z)$ converges also topologically to this element (see [2]).

(2) On the set $C(Y, Z)$ there exists the greatest splitting topology (see [2]).

(3) The intersection of all admissible topologies coincides with the greatest splitting topology (see, for example, [17,5,8]).

(4) The compact-open topology, denoted here by t_{co}, is always splitting (see [1] and [6]) and, in general, does not coincide with the greatest splitting topology (see [2]). For a regular locally compact space Y the topology t_{co} is always admissible and, therefore, coincides with the greatest splitting topology (see [6,1,2]).

(5) The Isbell topology is always splitting (see, for example, [15,14,18]) and, in general, does not coincide with the greatest splitting topology (see [5] and [9]). For a corecompact space Y (see, for example, [10]) the topology t_{Is} is always admissible and, therefore, coincides with the greatest splitting topology (see [14] and [18]).

(6) If Z is a Hausdorff space, then the topologies t_{co} and t_{Is} are Hausdorff (see [4] and [15]).

A space Y is called *Z-harmonic* if the compact-open topology coincides with the greatest splitting topology on $C(Y, Z)$. If Y is Z-harmonic for every space Z, then Y is called *harmonic* (see [8]).

A space Y is called *Z-concordant* if the Isbell topology coincides with the greatest splitting topology on $C(Y, Z)$. If Y is Z-concordant for every space Z, then Y is called *concordant* (see [8]). Let (Y, τ) be a space. Consider the subset $b \equiv \{ \mathrm{Int}(\mathrm{Cl}(U)) : U \in \tau \}$ of τ. The set b is a base for a topology on Y, denoted here by τ_{sr}. The space (Y, τ_{sr}) is called *semi-regular reduction* of (Y, τ). Obviously, $\tau_{sr} \subseteq \tau$. A topology τ is called *semi-regular* if $\tau = \tau_{sr}$. Therefore, a topology τ is semi-regular if and only if there exists a base for the topology τ such that $U = \mathrm{Int}(\mathrm{Cl}(U))$ for every element U of this base. It is easy to verify that: (α) τ_{sr} is a semi-regular topology and (β) any regular topology is semi-regular. We say that two spaces (Y, τ^0) and (Y, τ^1) have the same *semi-regular reduction* if.

$$\tau_{sr}^0 = \tau_{sr}^1$$

In this paper first we construct non-locally compact Hausdorff spaces Y such that the intersection of all admissible topologies on the set

$C(Y , Z)$, where Z is an arbitrary regular space, is admissible. Furthermore, for a Hausdorff splitting topology t on $C(Y , Z)$ we find sufficient conditions in order that t to be distinct from the greatest splitting topology. Using this result, we construct some concrete non-locally compact spaces Y such that the compact-open topology on $C(Y , Z)$, where Z is a Hausdorff space, is distinct from the greatest splitting topology. Finally, we give some open problems.

II. THE GREATEST SPLITTING AND ADMISSIBLE TOPOLOGIES

The following proposition is easily proved.

Proposition 2.1. *Let* (Y ,τ^0), (Y ,τ^1), *and* Z *be spaces such that*

$$C((Y,\tau^0),Z) = C((Y,\tau^1),Z)$$

72 *and for every space* X,

$$C(X\times(Y,\tau^0),Z) = C(X\times(Y,\tau^1),Z).$$

Then, a topology t *on* $C((Y ,\tau^0), Z)$ *is splitting (respectively, admissible) if and only if* t *is splitting (respectively, admissible) on* $C((Y ,\tau^1), Z)$.

Proposition 2.2. *(See [16, p. 85].) Let* (Y ,τ^0), (Y ,τ^1) *be two spaces with the same semi-regular reduction and* Z *a regular space. Then,*

$$C((Y,\tau^0),Z) = C((Y,\tau^1),Z)$$

and, therefore, for every space x,

$$C(X\times(Y,\tau^0),Z) = C(X\times(Y,\tau^1),Z).$$

Propositions 2.1 and 2.2 imply the following consequence.

Corollary 2.3. *Let* (Y ,τ^0), (Y ,τ^1) *be two spaces with the same semi-regular reduction and* Z *a regular space. Then, a topology* t *on* $C((Y ,\tau^0), Z)$ *is splitting (respectively, admissible) if and only if* t *is splitting (respectively, admissible) on* $C((Y ,\tau^1), Z)$. *Therefore, the intersection of all admissible topologies on* $C((Y ,\tau^0), Z)$ *is admissible if and only if the intersection of all admissible topologies on* $C((Y ,\tau^1), Z)$ *is admissible.*

Proposition 2.4. *Let* Y *be a space whose semi-regular reduction coincides with the semi-regular reduction of a regular locally compact space and* Z *an arbitrary regular space. Then, the intersection of all admissible topologies on* $C(Y , Z)$ *is admissible and, therefore, the greatest splitting topology is admissible.*

Proof. It suffices to find a topology on $C(Y , Z)$ which is simultaneously splitting and admissible. Let Y^{rlc} be the regular locally compact space whose semi-regular reduction coincides with the semi-regular reduction of Y . By Proposition 2.2 we have $C(Y^{rlc}, Z) = C(Y , Z)$. On the set $C(Y^{rlc}, Z)$ the compact-open topology t_{co} is splitting (since the compact-open topology is always splitting) and admissible (since Y^{rlc} is a regular locally compact space). By Corollary 2.3 the compact-open topology on $C(Y , Z)$ is also splitting and admissible proving the proposition. The following proposition is a generalization of Proposition 2.4.

Proposition 2.5. *Let* Y *be a space whose semi-regular reduction coincides with the semi-regular reduction of a core compact space and* Z *an arbitrary regular space. Then, the intersection of all admissible topologies on* $C(Y , Z)$ *is admissible and, therefore, the greatest splitting topology is admissible.*

Proof. As in the proof of the preceding proposition, it suffices to find a topology on $C(Y , Z)$ which is simultaneously splitting and admissible. Let Y^{cs} be the core compact space whose semi-regular reduction coincides with the semi-regular reduction of Y . By Proposition 2.2 we have $C(Y^{cs}, Z) = C(Y , Z)$. On the set $C(Y^{cs}, Z)$ the Isbell topology is splitting (since the Isbell topology is always splitting) and admissible (since Y^{cs} is a corecompact space). By Corollary 2.3 the Isbell topology on $C(Y , Z)$ is also splitting and admissible proving the proposition.

Similarly to the above two propositions we can prove the following propositions.

Proposition 2.6. *Let* Y *be a space whose semi-regular reduction coincides with the semi-regular reduction of a harmonic space* Y^h *and* Z *an arbitrary regular space (and, therefore,* $C(Y , Z) =$

$C(Y^h, Z)$). Then, the greatest splitting topology on $C(Y, Z)$ is the compact open topology on $C(Y^h, Z)$.

Proposition 2.7. *Let Y be a space whose semi-regular reduction coincides with the semi-regular reduction of a concordant space Y^{con} and Z an arbitrary regular space (and, therefore, $C(Y, Z) = C(Y^{con}, Z)$). Then, the greatest splitting topology on $C(Y, Z)$ is the Isbell topology on $C(Y^{con}, Z)$.*

Example 2.8.

(1) Let $[0,1]$ be the closed interval of the real line and let $M = \{1/n: n \in \omega\}$. Denote by Y the closed interval $[0,1]$ with the topology τ^1 for which the family $\tau^0 \cup \{Y \setminus M\}$, where τ^0 is the usual topology of $[0,1]$, compose a subbase. Obviously, the semi-regular reduction of the space $([0,1], \tau^1)$ is the space (regular compact space) $([0,1], \tau^0)$. By Proposition 2.4, the greatest splitting topology on the set $C(Y, [0,1])$ is admissible (see also Theorem 6.21 of [2]).

(2) Let (Y, τ^0) be an arbitrary space and M a subset of Y with the property that $\mathrm{Cl}(Y \setminus M) = Y$. On the set Y we consider the topology τ^1 for which the family $\tau^0 \cup \{Y \setminus M\}$ compose a subbase. It is easy to see that the spaces (Y, τ^0) and (Y, τ^1) have the same semi-regular reduction. By Proposition 2.5, for every corecompact space (Y, τ^0) and an arbitrary regular space Z, the greatest splitting topology on the set $C((Y, \tau^1), Z)$ is admissible. We note that the semi-regular reduction (X, τ_{sr}) of a corecompact space (X, τ) in general does not coincide with (X, τ).

III. ON THE SPLITTING TOPOLOGIES

Definition 3.1. We say that a net $\{f_\mu: \mu \in M\}$ in $C(Y, Z)$ has $f \in C(Y, Z)$ as *continuous cluster point* if for each $y \in Y$ and each neighborhood W of $f(y)$ in Z there is a neighborhood V of y in Y such that for each $\mu \in M$ there is $\mu_0 \in M$ with $\mu_0 \geqslant \mu$ such that $f_{\mu_0}(V) \subseteq W$.

Of course, if $\{f_\mu: {}^{\mu} \in M\}$ continuously converges to f, then f is a continuous cluster point of $\{f_\mu: {}^{\mu} \in M\}$.

Proposition 3.2. *Let t be a splitting Hausdorff topology on $C(Y, Z)$, where Y and Z are arbitrary spaces. If t is the greatest splitting topology, then every sequence $\{fi: i \in \omega\}$ in $C(Y, Z)$ which topologically converges to $f\infty \in C(Y, Z)$, has $f\infty$ as a continuous cluster point.*

Proof. Suppose that there exist a sequence $\{f_i: i \in \omega\}$ of elements of $C(Y, Z)$ and an element $f\infty$ of $C(Y, Z)$ such that:

(a) $\{f_i: i \in {}^{\omega}\} \quad \xrightarrow{t} \quad f\infty$ and

(b) neighborhood$f\infty$ is not a continuous cluster point of the sequenceW_0 of $f\infty(y_0)$ in Z such that for every open neighborhood$\{f^i: i \in {}^{\omega}\}$, that is there exist a point$U$ of $y0$ in Y there existsy^0 $_i(U') \in \omega$of Y and an open for which $f_i(U') \nsubseteq W_0$ for every $i \geqslant i(U')$.

To prove the proposition it suffices to show that t is not the greatest splitting topology.

We note that the set $F = \{f_i: i \in {}^{\omega}\}$ is infinite. Indeed, in the opposite case, there exists $k \in \omega$ such that $f_i = f_k$ for all elements i of an infinite subset ω' of ω. Since t is a Hausdorff topology and $\{f_i: i \in \omega'\} \xrightarrow{t} f\infty$ we have that $f\infty = f_i$ for every $i \in \omega$. This fact contradicts the above condition (b). Therefore, the set F is infinite and, without loss of generality, we can suppose that $f\infty \neq f_i$ for every $i \in \omega$.

We put $b = t \cup \{U \cap G: U \in t\}$, where $G = C(Y, Z) \setminus F$. It is easy to see that the intersection of two elements of b is an element of $=$ b. Let $t+\in$be the topology on $C(Y, Z)$ for which the set b is a base. Clearly, $t \subseteq t+$. Since $\{+f_i$, the set: $i \in {}^{\omega}\} - F \xrightarrow{t}$ is closed $f\infty$ and inf$_\infty C_{t+}(fY_i$,for every$Z$). Therefore,i ω, the set$t$ $= t+$. F is not closed in $C_t(Y, Z)$. On the other hand, by the definition of t

To prove that t is not the greatest splitting topology it suffices to show that the topology $t+$ is splitting. Let $\{g_\mu, \mu \in M\}$ be a net in $C(Y, Z)$ converging continuously to an element g of $C(Y, Z)$. We need to prove that $\{g_\mu, \mu \in M\} \xrightarrow{t+} g$, that is for every open neighborhood V of g in the space $C_{t+}(Y, Z)$ there exists an element$\mu' \in M$ such that $g\mu \in V$ for every $\mu \geqslant \mu$. Note that, since t is splitting, $\{g_\mu, {}^{\mu} \in M\} \xrightarrow{t} g$.

First, we consider the case $gf\mu' \in$such that $g\mu \in V^t$ for every$\mu \geqslant \mu$. Therefore, we can suppose that∞.

Let $V \in b$ be an neighborhood of$V = gV$in$\cap CGt+$,where(Y,Z).If$VV\in\in t$. In this case,t, then there exists$g \in/ F \cup \{ f\infty M\}$.

Since $\{ f_i\colon i \in \omega \} \longrightarrow f\infty$ and the space $C_t(Y , Z)$ is Hausdorff there exists an open neighborhood V_g of g such that $V_g \cap (F \cup \{ f_\infty \}) = \emptyset$ and, therefore, $V_g \subseteq G$. Then,

$$V_g \cap V = V_g \cap V' \cap G = V_g \cap V' \in t.$$

Therefore, there exists$\mu' \in M$ such that $g_\mu \in V_g \cap V' \subseteq V$ for all $\mu \geqslant \mu$

Now, we consider the case $g = f_\infty$. Suppose that the net $\{ g_\mu, \mu \in M \}$ does not converge to g in the space $C_{t^+}(Y , Z)$. Then, there exists $V \in b$ such that $g \in V$ and for every$\mu' \in M$ there exists$\mu \geqslant \mu'$ with $g_\mu \in/ V$. This fact implies that V is not an element of t and, therefore, $V = V \cap G$ where $V \in t$. Without loss of generality, we can suppose that $g_\mu \in/ V$ and $g_\mu \in V$ for every $\mu \in M$, which means that $\{ g_\mu\colon \mu \in M \} \subseteq C(Y , Z) \setminus G = F$.

Therefore, there is a map $\sigma : M \to \omega$ such that $g_\mu = f_{\sigma(\mu)}$, $\mu \in M$.

Let W_0 be the open neighborhood of $f\infty(y_0)$ considered in the above condition (b). Since the net $\{ g_\mu,\mu \in M \}$ continuously converges to f_∞ there exist an open neighborhood U_0 of y_0 in Y and an element $\mu_0 \in M$ such that $g_\mu(U_0) \subseteq W_0$ for every $\mu \geqslant \mu_0$. On the other hand, by condition (b), for the set U_0 there exists $i(U_0) = i_0 \in \omega$ such that $f_i(U_0) \nsubseteq W_0$ for every i i_0. This fact implies that $\sigma(\mu) < i_0$ for every$\mu \geqslant \mu_0$.

Let V_0 be an open neighborhood of $f\infty$ in $C_t(Y , Z)$ such that $f_i \in/ V_0$ for every $i < i_0$. Then, $g_\mu = f_{\sigma(\mu)} \in/ V_0$ for every $\mu \geqslant \mu_0$ and, therefore, the net $\{g_\mu\colon \mu \geqslant \mu_0\}$ does not converge to $f_\infty = g$ in the space $C_t(Y , Z)$ which is a contradiction

proving that the net $\{ g_\mu, \mu \in M \}$ converges to g in the space $C_{t^+}(Y , Z)$.

Thus, the topology $t+$ is splitting completing the proof of the proposition.74

Corollary 3.3. *Let Y be an arbitrary space. If there exist a Hausdorff space Z and a sequence $\{ f_i\colon i \in \omega \}$ of elements of $C_{tco}(Y , Z)$ which converges to $f_\infty \in C_{tco}(Y , Z)$ such that $f\infty$ is not a continuous cluster point, then Y is not harmonic.*

Corollary 3.4. *Let Y be an arbitrary space. If there exist a Hausdorff space Z and a sequence $\{ f_i\colon i \in \omega \}$ of elements of $C_{tIs}(Y , Z)$ which converges to $f_\infty \in C_{tIs}(Y , Z)$ such that $f\infty$ is not a continuous cluster point, then Y is not concordant.*

The following example gives a method of construction of nonharmonic spaces.

Example 3.5. Let $Y_i, i \in \omega$, be a family of mutually proving that $\{ g_\mu, \mu \in M, \} \to g$. disjoin Hausdorff spaces. Suppose that for every $i \in \omega$ there exists a filter Fi of non-empty open sets of Y_i with the property that $Y_i \setminus K \in$ Fi for every compact subset K of Y_i. On the set

$$Y = \left(\bigcup\{Y_i\colon i \in \omega\}\right) \bigcup \{\infty\},$$

where ∞ is a symbol, we consider a topology for which a subset V of Y is open if and only if:

(α) $V \cap Y_i$ is open in Y_i for all $i \in \omega$, and

(β) in the case where $\infty \in V$, there exists a finite subset s of ω s

We note that the space Y has the properties:

(1) Y_i is simultaneously open and closed subspace of Y .

(2) If K is a compact subset of Y , then there exists a finite subset s of ω such that $K \subseteq \cup\{ Y_i\colon i \in s \} \cup \{\infty\}$.

We shall prove that Y is not harmonic. Let Z be an arbitrary Hausdorff space containing two distinct points a and b.

Consider the sequence $\{ f_i, i \in \omega \}$ of maps of Y into Z for which $f_i(y) = b$ if $y \in Y_i$ and $f_i(y) = a$ if $y \in Y \setminus Y_i$. Let also $f\infty$ be the element of $C(Y , Z)$ defined by condition $f\infty(y) = a$ for all $y \in Y$. Using the above properties (1) and (2) one can prove tha t the maps f_i are elements of $C(Y , Z)$ and the sequence $\{ f_i, i \in \omega \}$ converges to $f\infty$ in the compact-open topology. By Corollary 3.3 it suffices to prove that $f\infty$ is not a continuous cluster point of the sequence $\{ f_i, i \in \omega \}$. Let W_0 be an open neighborhood of a which does not contain the point b. Consider an arbitrary open neighborhood U' of ∞ in Y . Then, there exists a finite subset s of ω such that $U \cap Y_i = \emptyset$ for every $i \in \omega \setminus s$. Setting $i(U') = \max\{i\colon i \in s\}$ we have that $f_i(U') \nsubseteq W_0$ for every i $i(U')$ proving that $f\infty$ is not a continuous cluster point.

Remark 3.6. The above example can be considered as a generalization of the space *Y* considered in Theorem 5.3 of [2].

IV. SOME OPEN PROBLEMS

Problem 4.1. Let P be a class of spaces *Y* having the same semi-regular reduction and *Z* a regular space. By Proposition 2.2 and Corollary 2.3 the set $C(Y, Z)$ and the greatest splitting topology on this set are independent of the elements *Y* of P.

(1) Is the compact-open topology on $C(Y, Z)$ independent of the elements *Y* of P?

(2) Is the Isbell topology on $C(Y, Z)$ independent of the elements *Y* of P?

(3) Suppose that P contains an element which is *Z*-harmonic (respectively, harmonic). Is any element of P *Z*-harmonic (respectively, harmonic)?

(4) Suppose that P contains an element which is *Z*-concordant (respectively, concordant). Is any element of P *Z*concordant (respectively, concordant)?

Problem 4.2. It is known that the compact-open topology does not coincide with the greatest splitting topology on the set $C(N^\omega, N)$ (see [5]), as well as, on the set $C(R^\omega, R)$ (see [9]), where *N* is the set of natural numbers with the discrete topology and *R* is the set of real numbers with the usual topology.

(1) Suppose that the space *Z* is the space *N* or the space *R*. Can we find a sequence $\{ f_i : i \in \omega \}$ of elements of $C(Z^\omega, Z)$converges in the compact-open topology to an element $f_\infty \in C(Z^\omega, Z)$ such that $f\infty$ is not a continuous cluster point?

(2) Let *Y* and *Z* be two spaces such that the compact-open (respectively, the Isbell) topology on $C(Y, Z)$ is not the greatest splitting topology. Under what (internal) conditions on *Y* and *Z* are the conditions of Corollary 3.3 (respectively, Corollary 3.4) satisfied?

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