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Fuzzy AHP and its application to select best factor for the success of e-commerce

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ABSTRACT
Analytic Hierarchy process is one of the methods of Multi Criteria Decision making. In AHP uncertainty of decision maker cannot be expressed clearly. So AHP is studied in Fuzzy environment and Fuzzy AHP is developed. In this paper another method of Fuzzy AHP is introduced. In this method comparison matrices are represented as triangular fuzzy number. From this comparison matrices the priority vectors are evaluated using Nearest Weight Interval Approximation and score function. This method is applied to the problem of identifying best success factor for the e-commerce

Keywords: Fuzzy AHP, Triangular fuzzy number, $\propto -cut$, Comparison matrices.

I. INTRODUCTION

Decision making is very important in day today's life. It is a cognitive process in which optimal option is selected from available alternatives based on gathered information. Multi Criteria Decision Making is one of the process of decision making in which best alternative is selected from the available alternatives based on multi criteria. MCDM methods consist of an extensive variety methodologies such as AHP. ELECTRE, TOPSIS, PROMTHEE, VIKOR etc.

Analytic Hierarchy Process (AHP) is a very powerful tool introduced by Saaty in 1950 [1] to handle problems which are complex and not structured properly. In this, a complex problem is decomposed into multi-level hierarchical structure which contains goal, criteria, sub-criteria and alternatives. In AHP decision maker's uncertainty and ambiguity cannot be expressed. Fuzzy set theory introduced by Zadeh (1965) [2] is an effective tool to express ambiguity and uncertainty. To express the decision maker's vagueness in AHP, it is incorporated with Fuzzy set theory and Fuzzy AHP is introduced.

FAHP is extensively studied by many authors. Vaan Laarhoven and Pedryz [3] studied FAHP using Logarithmic Least square method for deriving priorities. C.G.E Boender et al [4] improved on normalized procedure of former. L.Mikhailov [5] introduced Alpha cut method to derive priorities from fuzzy pairwise comparison matrices. The most useful and applied method is introduced by Chang [6] in which extent analysis method is used to derive priorities.

E-commerce is a method of buying and selling goods and services online. E-commerce began in 1960's when companies used to interchange the documents through electronic. In 1994 first transaction was made. The COVID-19 pandemic of 2020 caused e-commerce to spike significantly.

To succeed in e-commerce the companies should focus on key factors such as brand name, customer satisfaction, etc. Many researchers have researched on the key factors for success of e-commerce. Kong and Liu [7] tried to find the best factor of e-commerce used FAHP method.

In this paper Fuzzy AHP is used to identify the best factor for the success of e-commerce. To do so this paper has structured as follows.

2. Preliminaries:

Definition: 2.1 Fuzzy set

Let X be a nonempty set. A fuzzy set $\tilde{A}$ in X is characterized by its membership function

$$\mu_{\tilde{A}} : X \to [0,1]$$

and $\mu_{\tilde{A}}(x)$ is interpreted as the degree of membership of element x in fuzzy set A for each $x \in X$. A fuzzy set $\tilde{A}$ is written as $A = \{(x, \mu_{\tilde{A}}(x)) / x \in X\}$

Definition: 2.2 α -level set

An α -level set of a fuzzy set $\tilde{A}$ of X is a non-fuzzy set denoted by ${}^{\alpha}\tilde{A}$ and it is defined as ${}^{\alpha}\tilde{A} = \{x / \tilde{A}(x) \geq \alpha\}$

Definition: 2.3 Fuzzy Numbers:

A fuzzy number is a fuzzy set $\tilde{A}$ on the real line $\square$ with the following properties:

1. $\tilde{A}$ must be a normal fuzzy set.
2. α -cut of $\tilde{A}$ is a closed interval for every $\alpha \in [0,1]$
3. The support of $\tilde{A}$ is bounded.

Many types of fuzzy numbers are defined in the literature. Among these Triangular fuzzy number and Trapezoidal fuzzy number are two of them.

Definition: 2.4 Triangular Fuzzy Number:

A fuzzy number $\tilde{A}$ on $\square$ to be Triangular Fuzzy Number if its membership function $\mu_{\tilde{A}} : \square \to [0,1]$ is equal to

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-l}{m-l} & l \le x \le m \\ \dfrac{u-x}{u-m} & m \le x \le u \\ 0 & otherwise \end{cases}$$

Where l and u represent the lower and upper bounds of a fuzzy number $\tilde{A}$ and m is the median value. The Triangular Fuzzy Number is denoted as $\tilde{A} = (l, m, u)$.

Definition: 2.5 Weight function

A weighting function is a function as $f = (f, \check{f}) : ([0,1],[0,1]) \to (R, R)$ are non negative and uniformly increasing and satisfy the following condition: $\int_0^1 f(\alpha)d\alpha = \int_0^1 \check{f}(\alpha)d\alpha$

Definition: 2.6 Nearest Weighted Interval Approximation

Let $\check{A} = (l, m, n)$ be a triangular fuzzy number and let $f(a) = (ya^{y-1}, ya^{y-1})$ be a weighting function. Then NWIA $(\check{A}) = \left[\frac{l+ym}{y+1}, \frac{ym+n}{y+1}\right]$.

Definition: 2.7 Score function of Nearest Weighted Interval Approximation

Let $A = [a, b]$ be a Nearest Weighted Interval Approximation of $\check{A}$ then the score function of A is defined as $S(A) = \frac{a^2+b^2}{2}$

II. PROCEDURE FOR FAHP

Step: 1 The problem is constructed as hierarchical structure which contains goal, criteria , sub criteria and alternatives.

Step: 2 The preferences of decision maker is given in the form of Triangular fuzzy number with the help of preference matrix given in the below table.

Linguistic variable	Fuzzy number	Reciprocal Fuzzy number
Equally important	(0.5, 0.5, 0.5)	
Moderately important	(0.5,0.6,0.7)	(0.3,0.4,0.5)

Strongly important	(0.6,0.7,0.8)	(0.2,0.3,0.4)
Very strongly important	(0.7,0.8,0.9)	(0.1,0.2,0.3)
Absolutely important	(0.8,0.875,0.95)	(0.05,0.125,0.2)

Step: 3 This Triangular Fuzzy number is transformed into nearest weighted interval approximation which is an interval value fuzzy set.

Step:4 From this interval valued fuzzy number the score function is evaluated for all the elements of comparison matrix .

Step: 4 Determine the priority vectors using $w_i = \frac{\prod_{i=1}^{n} \widetilde{A_{ij}}}{\prod_{i=1}^{n} \prod_{j=1}^{m} \widetilde{A_{ij}}}$

for all the comparison matrices.

Step: 5 All the weights from the lowest level of the hierarchy to the highest level of the hierarchy are combined and ranked.

III. SUCCESS FACTORS OF E-COMMERCE:

The products which are manufactured all over the globe can be brought through a online platform and distributed to various placed is called as e-commerce. Success of e-commerce companies based on many factors and sub factors which is given in the following diagram.

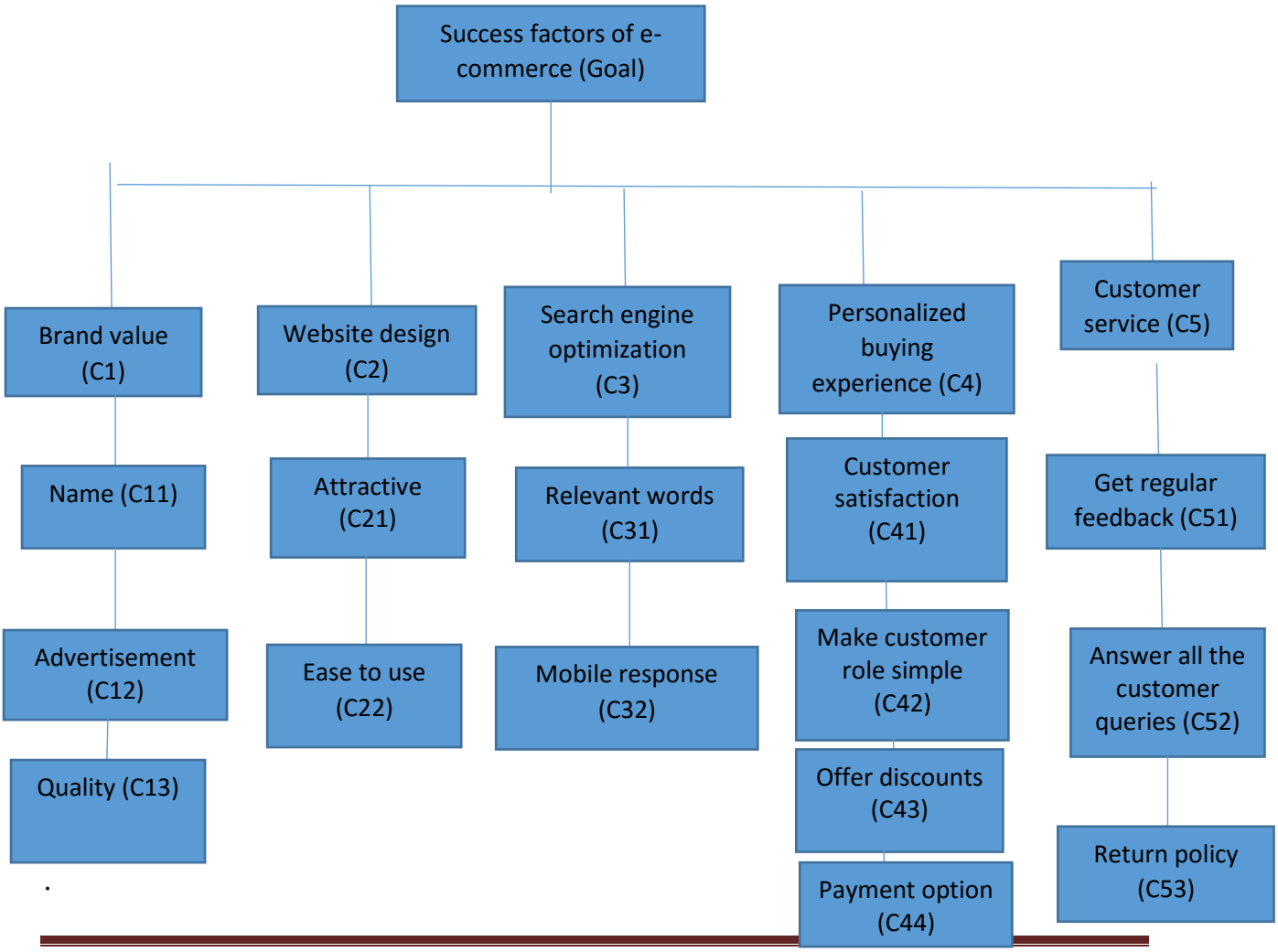

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Diagram 1 (hierarchical structure of the problem)

Table-2.1 Criteria comparison matrix

	C1	C2	C3	C4	C5
C1	Equally	Moderately	moderately	Strongly	Very strongly
C2		Equally	Moderately	strongly	Very strongly
C3			Equally	strongly	equally
C4				Equally	Strongly
C5					Equally

Table-2.2 Criteria comparison matrix

	C1	C2	C3	C4	C5
C1	(0.5, 0.5, 0.5)	(0.5,0.6,0.7)	(0.5,0.6,0.7)	(0.6,0.7,0.8)	(0.7,0.8,0.9)
C2	(0.3,0.4,0.5)	(0.5, 0.5, 0.5)	(0.5,0.6,0.7)	(0.6,0.7,0.8)	(0.7,0.8,0.9)
C3	(0.3,0.4,0.5)	(0.3,0.4,0.5)	(0.5, 0.5, 0.5)	(0.6,0.7,0.8)	(0.5, 0.5, 0.5)
C4	(0.2,0.3,0.4)	(0.2,0.3,0.4)	(0.2,0.3,0.4)	(0.5, 0.5, 0.5)	(0.6,0.7,0.8)
C5	(0.1,0.2,0.3)	(0.1,0.2,0.3)	(0.5,0.5,0.5)	(0.2,0.3,0.4)	(0.5, 0.5, 0.5)

Table-2.3 Nearest weighted interval approximation

	C1	C2	C3	C4	C5
C1	[0.5, 0.5]	[0.55,0.65]	[0.55,0.65]	[0.65,0.75]	[0.75,0.85]
C2	[0.45,0.55]	[0.5, 0.5]	[0.55,0.65]	[0.65,0.75]	[0.75,0.85]
C3	[0.45,0.55]	[0.45,0.55]	[0.5, 0.5]	[0.65,0.75]	[0.5, 0.5]
C4	[0.25,0.35]	[0.25,0.35]	[0.25,0.35]	[0.5, 0.5]	[0.65,0.75]
C5	[0.15,0.25]	[0.15,0.25]	[0.5, 0.5]	[0.25,0.35]	([0.5, 0.5]

Table-2.4 Score function

	C1	C2	C3	C4	C5
C1	0.25	0.3625	0.3625	0.4925	0.6425
C2	0.2525	0.25	0.3625	0.4925	0.6425
C3	0.2525	0.2525	0.25	0.4925	0.25

C4	0.0925	0.0925	0.0925	0.25	0.4925
C5	0.0425	0.0425	0.5	0.0925	0.25

Table-2.5 Criteria weights

	C1	C2	C3	C4	C5
Weights	0.1178	0.1323	0.2076	0.241	0.3016

Likewise weight vectors for all the criteria and sub criteria are calculated and given in the following table

Criteria	Criterion weights	Sub criteria		Final weight values	
		Sub criteria	Weights	Sub criteria weights	Criteria weights
C_1	0.1178	C_{11}	0.1	0.1178	0.20035
		C_{12}	0.4	0.04721	
		C_{13}	0.3	0.03534	
C_2	0.1323	C_{21}	0.35	0.0463	0.132295
		C_{22}	0.65	0.085995	
C_3	0.2076	C_{31}	0.45	0.09342	0.2066
		C_{32}	0.55	0.11318	
C_4	0.241	C_{41}	0.4	0.0964	0.241
		C_{42}	0.15	0.03615	
		C_{43}	0.15	0.03615	
		C_{44}	0.3	0.0723	
C_5	0.301	C_{51}	0.1	0.0301	0.3073
		C_{52}	0.2	0.0602	
		C_{53}	0.7	0.217	

IV. RESULTS AND DISCUSSION

The final weight vectors for the factors are 0.2003, 0.132295, 0.2066, 0.241, and 0.3073.

From this it can be concluded the customer service is the most important factor for success of e-commerce companies. In this paper another method of Fuzzy analytic hierarchy process is introduced in which Nearest Weight Interval Approximation and score function is used to derive priority vectors from triangular fuzzy comparison matrices which yields good results. This method is applied to identify best success factor are e-commerce in which the companies has to concentrate..

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