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A Brief Study on Formulation of Some Transportation Problem in Linear Programming

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ABSTRACT
The Transportation problem is one of the ultimate optimization problems in the branch of optimization or operation research. The transportation problem mostly focuses on the best possible way a product manufactured at different supply origin can be transported to a various demand destination. The objective in a Transportation Problem is to fully gratify the destination necessities with in the functioning manufacture capacity constraints at minimum feasible cost. The construction of transportation problem comprises a large number of transport routs from numerous supply origin to numerous demand destinations. The coefficient of the objective function can characterise transportation cost, time, profit, product defectiveness. The objective is to find the number units dispatched from on origin to a destination, with in the limited quantity of goods or services presented at each supply origin at the minimum transportation cost or time.
The basic concept explain in the journal is applied to minimize the total transportation cost of transporting a product from supply origin distributions. However, it can also be applied to the maximization of some total value or utility.
Keywords:- Transportation Problem, Fuzzy Transportation Problem, Solid Transportation Problem, Multi-objective Transportation Problem

I. INTRODUCTION

The simple transportation problem originally develop by Hitchcok [1] has also offered a study entitled. This demonstration has reflected the first significant involvement to the result of transportation problem. Kantorovich [2] Studied on continuous form of the problem and further with Gavurian they studied the capacitated transportation problem. Koopmans [3] has presented “optimum utilization of the transportation system” The transportation problem was modelled as a standard linear programming problem and efficient solution Transportation problem was obtained using the simplex method in 1947. George B. Dantzing [4] applied the concept of linear programming in solving the transportation models; it could be solved for optimally as an answer to complex business problem in 1951. William [5] applied the decomposition principle fot the solution of the simple transportation problem. Shetty [6] has considered a convex production cost at each supply centre and formulated an algorithm to solve transportation problem. Charnes and cooper [7] provided an alternative way of determining the simplex-method information by developing the sleeping stone method. Dantzing [8] used the primal simplex transportation method and obtained an initial basic feasible solution for the transportation problem by using the northwest corner rule, row minima, column minima, matrix minima or the vogel’s approximation method. The modified distribution method is useful for the optimal soluation for the transportation problem.
Swarup [9] developed a technique, similar to transportation technique in linear programming to minimize a locally indefinite quadratic function.
Sharp et.al. [10] Developed an algorithm for an optimal solution of the production transportation problem. Sharma [11] has proposed a new heuristic approach for accomplishment good and starting solutions for dual based approaches for solving transportation problem. Kakuchi [12] suggested that in many problems of transportation engineering and planning, the observed or resultant value of the variables are inexact, nevertheless the variable themselves must gratify the set of rigid relationship uttered by physical principle. For each of many possible sets of values the gratify he relationship the lowest membership grade is checked and the set, whose lowest membership grade is the highest, is chosen as the best set of value for the problem.

II. MATHEMATICAL FORMULATION OF SINGLE OBJECTIVE TRANSPORTATION PROBLEM.

2.1 Single objective optimization problem: -

A single objective optimization problem finds the maximum or minimum value of a given objective function, subjected to a given set of constraints that must be satisfied by single objective optimization problem solution

$$\text{Max (or min) } f(y)$$

Subject to the constraint:

$$\boldsymbol{x} \in \boldsymbol{\Omega}$$

where f is a given objective function from a general multidimensional space T^q to the set of real T, and $\Omega = (y/h_i(y)\ (\leq, =, \geq)\ d_i,\ i = 1,2,\ldots\ldots\ldots\ldots p)$ is a subset of T^q defined by various conditions called 'constrains one of the main goals of optimization is to find an efficient algorithms to obtain the optimal solution, that is, to find a vector $y \in \Omega$ that optimization (maximizes or minimizes) the objective function f among the set of all feasible salutation. Depending on the nature of the function involved in problem of Max (or Min) $f(y)$, $y \in \Omega$.

2.2 Single objective internal transportation problem: -

The invention of interval transportation problem is the problem of optimizing interval valued objective function with interval constraints.

Formulation of interval transportation problem

$$\text{Minimize } Y = \sum_{i=1}^{p} \sum_{j=1}^{q} [D_{K_{ij}}, D_{T_{ij}}]\, y_{ij}$$

Subject to the constraints $\sum_{j=1}^{q} y_{ij} = [c_{K_i}, c_{T_i}]$, i=1, 2 …. p

$$\sum_{i=1}^{p} x_{ij} = \left[d_{K_j}, d_{T_j}\right],\ j = 1,2, \ldots \ldots . q$$

$$y_{ij} \geq 0, \forall i, j$$

With

$$\sum_{i=1}^{p} c_{K_i} = \sum_{j=1}^{q} d_{K_j} \text{ and } \sum_{i=1}^{p} c_{T_i} = \sum_{j=1}^{q} d_{T_j}$$

Where $[D_{K_{ij}}, D_{T_{ij}}]$ is an interval demonstrating the uncertain cost for the transportation problem and it can characterise transport time, quantity of product delivered, under used capacity, etc. the source parameter lies among left limit $c_{K_i}$and right limit c_{T_i}. Similarly, destination parameter lies between left limit d_{K_i} and right limit d_{T_i}.

2.3 Single objective fuzzy transportation problem: -

Formulation of fuzzy transportation Problem

Minimize

$$Y = \sum_{i=1}^{p} \sum_{j=1}^{q} \tilde{e}_{ij}\, y_{ij},$$

Subject to the constraints:

$$\sum_{j=1}^{q} y_{ij} = \tilde{c}_I \qquad i = 1, 2 \ldots\ldots\ldots\ldots p$$

$$\sum_{i=1}^{p} y_{ij} = \tilde{d}_j \qquad j = 1, 2 \ldots\ldots\ldots\ldots n$$

$$y_{ij} \geq 0,\ \forall_{i,j}$$

Where,

$\tilde{c}_i$ be the fuzzy supply of the product i^{th} source

$\tilde{d}_i$ be the fuzzy demand of the product at j^{th} destination

$\tilde{e}_{ij}$ be the fuzzy transportation cost for unit quantity of the product from i^{th} source of j^{th} destination.

y_{ij} be the quantity of the product that should be shipped from i^{th} source to j^{th} destination to minimize the total fuzzy transportation cost.

2.4 Single objective solid transportation problem: -
The mathematical formulation of single objective solid transportation problem
Minimize:

$$Y = \sum_{i=1}^{p} \sum_{j=1}^{q} \sum_{t=1}^{S} e_{ijt}\, y_{ijt}$$

Subject to the constraints:

$$\sum_{j=1}^{q} \sum_{b=1}^{s} y_{ijt} = \tilde{c}_i,\ i=1, 2\ldots\ldots\ldots p$$

$$\sum_{j=1}^{p} \sum_{t=1}^{s} y_{ijt} = d_j,\ j= 1, 2\ldots\ldots\ldots..q$$

$$\sum_{i=1}^{p} \sum_{j=1}^{q} y_{ijt} = f_t,\ t= 1, 2\ldots\ldots\ldots..S$$

$$y_{ijt} \geq 0,\ \forall_{i,j,t}$$

Where,

C_i represents supply of the product at i^{th} source.

d_j be the demand of the product at j^{th} destination.

f_t represents the transportation capacity of the conveyance *t*.

c_{ijt} is a penalty associated with transportation of a unit of the product from source *i* to destination *j* by means of the t^{th} conveyance

y_{ijt} represents the unknown quantity to be transported from origin to destination by means of the t^{th} conveyance.

The solid transportation problem was first identified by schell [13] verdegay [14] considered two type of uncertain solid transportation problem, one with interval numbers and other with fuzzy numbers.

2.5 Single objective fuzzy solid transportation problem: -
A fuzzy solid transportation problem of minimizing objective function is formulation as follow.
Minimize:

$$Y = \sum_{i=1}^{p} \sum_{j=1}^{q} \sum_{t=1}^{s} \tilde{e}_{ijt}\, y_{ijt}$$

Subject to the constraints

$$\sum_{j=1}^{q} \sum_{t=1}^{s} y_{ijt} = \tilde{c}_i\ ,\ i = 1, 2\ldots\ldots..p$$

$$\sum_{i=1}^{p} \sum_{t=1}^{s} y_{ijt} = \tilde{d}_j\ ,\ j = 1, 2\ldots\ldots.\ n$$

$$\sum_{i=1}^{p} \sum_{j=1}^{q} y_{ijt} = \tilde{f}_t\ ,\ t = 1, 2\ldots\ldots..S$$

$$y_{ijt} \geq 0, \text{ for all } i, j, t$$

Where,

$\tilde{c}_i$ represents fuzzy supply of the product at i^{th} source.

$\tilde{d}_j$ be the fuzzy demand of the product at j^{th} destination.

$\tilde{f}_t$ represent the transportation capacity of conveyance *t*.

$\tilde{e}_{ijt}$ is a fuzzy penalty associated with transportation of a unit of the product from source *i* to destination *j* by means of t^{th} conveyance.

y_{ijt} represents the unknown quantity to be transported from origin to destination by means of the t^{th}conveyance.

2.6 Single objective fuzzy fixed charge solid transportation problem:-
A mathematical model of fuzzy charge solid transportation problem is formulated as follows.
Minimize:

$$\mathrm{Y} = \sum_{i=1}^{p} \sum_{j=1}^{q} \sum_{t=1}^{s} (\tilde{e}_{ijt} y_{ijt} + \vartheta_{ijt}\, u_{ijt})$$

Subject to the constraints

$$\sum_{j=1}^{q} \sum_{t=1}^{s} y_{ijt} \leq \tilde{c}_i \ ,\ i = 1, 2 \ldots\ldots p$$

$$\sum_{i=1}^{p} \sum_{t=1}^{s} y_{ijt} \geq \tilde{d}_j \ ,\ i = 1, 2 \ldots\ldots q$$

$$\sum_{i=1}^{p} \sum_{j=1}^{q} y_{ijt} \leq \tilde{f}_t \ ,\ t = 1, 2 \ldots\ldots S$$

$$y_{ijt} \geq 0, \forall\, i, j, l$$

$$u_{ijt} = \begin{cases} 1; & if\ y_{ijt > 0} \\ 0; & otherwise. \end{cases}$$

Where,

$\tilde{c}_i$ = fuzzy supply of the product at i^{th} source.

$\tilde{d}_j$ = fuzzy demand of the product at j^{th} destination.

$\tilde{\vartheta}_{ijt}$ = the fixed charge with respect to transportation active from source *i* to *j* destination by conveyance *t*.

$\tilde{e}_t$ = the fuzzy transportation capacity of conveyance *t*.

$\tilde{x}_{ijt}$ = the unknown quantity to be transported from origin to destination by means of the t^{th} conveyance.

III. MATHEMATICAL FORMULATION OF MULTI-OBJECTIVE TRANSPORTATION PROBLEM.

3.1 Multi-Objective Optimization Problem: -
Multi-Objective optimization model also known as multi-objective programming, vector optimization problem, multi-criteria optimization or pareto optimization. Multi-Objective optimization model is used for mathematical optimization problem comprising more than one objective function to be optimized simultaneously. It is multiple criteria decision-making and useful in many fields of science, engineering, economics and logistic where optimum decision need to be occupied in the optimization occurrence of trade-offs among two or more conflicting objectives.
Max (or Min) θ (*y*) = (θ_1 (*y*), $\theta(y)$ ... ...$\ldots\theta_t$(*y*))

Subject to the constraint:

$$y \in \Omega$$

Where: $\Omega \rightarrow T^t$ is a given vector function consisting of t objective function to be maximized or minimized.

3.2 Multi-objective Transportation Problem:-
The mathematical structure of transportation problem with l objectives, p sources and q destinations
Minimize:

$$\mathrm{Y}_l = \sum_{i=1}^{p} \sum_{j=1}^{q} e_{ij}^{l}\, y_{ij},\ l = 1 \ldots\ldots J$$

Subject to the constraints

$$\sum_{j=1}^{q} y_{ij} = c_i \text{ , } i = 1, 2\ldots\ldots\ldots p$$

$$\sum_{i=1}^{p} y_{ij} = d_j, j = 1, 2\ldots\ldots\ldots q$$

$$y_{ij} \geq 0, \forall_{i,j}$$

3.3 multi-objective interval transportation problem: -

The mathematical formulation:

Minimize:

$$Y^l = \sum_{i=1}^{p} \sum_{j=1}^{q} [D_{K_{ij}}^{l}, D_{T_{ij}}^{l}]\, y_{ij}$$

Subject to the constraints:

$$\sum_{j=1}^{q} y_{ij} = [c_{K_i}, c_{T\,Ti}], i = 1, 2\ldots\ldots\ldots\ldots p$$

$$\sum_{i=1}^{p} y_{ij} = [d_{K_j}, d_{T_j}], j = 1, 2\ldots\ldots\ldots\ldots n$$

$$y_{ij} \geq 0, \forall_{i,j}$$

With $\sum_{i=1}^{p} c_{K_i} = \sum_{j=1}^{q} d_{K_j}$ and $\sum_{i=1}^{p} c_{T_i} = \sum_{j=1}^{q} d_{T_j}$

Where, the source parameter between left limit c_{K_i} and right limitc_{T_i}.

Similarly, destination between left limit d_{K_j} and right limit d_{T_j} and $[D_{K_{ij}}^{l}, D_{T_{ij}}^{l}]$, ($k$ = 1, 2……J) is an interval indicating the uncertain cost the transportation problem.

3.4 Fuzzy multi-objective transportation problem: -

Minimize:

$$\tilde{Y}_l = \sum_{i=1}^{p} \sum_{j=1}^{q} (\tilde{D}_{ij}^{l}) y_{ij}$$

For l = 1, 2.

Minimize $\tilde{Y}_1 = \sum_{i=1}^{p} \sum_{j=1}^{q} (\tilde{D}_{ij}^{l})\, y_{ij}$ and minimize $\tilde{Y}_2 = \sum_{i=1}^{p} \sum_{j=1}^{q} \tilde{D}_{ij}^{2}\, y_{ij}$

Subject to the constraints:

$$\sum_{j=1}^{q} y_{ij} \leq \tilde{c}_i \text{ , } i = 1, 2\ldots\ldots\ldots p$$

$$\sum_{i=1}^{p} y_{ij} = \tilde{d}_j \text{ , } j = 1, 2\ldots\ldots\ldots\ldots q$$

$$y_{ij} \geq 0, \forall i, j$$

Where,

y_{ij} = unit distributed from source i to destination j.

$\tilde{Y}_1$ = total distribution cost

$\tilde{Y}_2$ = total product impairment

$\tilde{D}_{ij}^{1}$ = distribution cost per unit delivered from source t to destination j.

$\tilde{D}_{ij}^{2}$ = product impairment during delivered from source i to destination.

$\tilde{c}_i$ = total available supply for each source i

$\tilde{d}_j$ = total demand of each destination j.

3.5 Multi-objective solid transportation problem:-
A multi objective solid transportation problem of minimizing objective function is formulated.
Minimize:

$$Y_l = \sum_{i=1}^{p} \sum_{j=1}^{n} \sum_{b=1}^{S} D_{ijt}^{l}\, y_{ijt}\ ,\ l= 1, 2\ldots\ldots J$$

Subject to the constraints:-

$$\sum_{j=1}^{q} \sum_{t=1}^{S} y_{ijt}=c_i\ ,\ i= 1, 2\ldots\ldots\ldots\ldots p$$

$$\sum_{i=1}^{p} \sum_{t=1}^{S} y_{ijt}=d_p\ ,\ j=1, 2\ldots\ldots\ldots\ldots q$$

$$\sum_{i=1}^{p} \sum_{j=1}^{q} y_{ijt} =f_t\ ,\ t = 1, 2\ldots\ldots\ldots S$$

$$y_{ijt} \geq 0,\ \forall_{i,j,t}$$

Where,

c_i represents supply of the product at i^{th} source.

d_j be the demand of the production at j^{th} destination

l represents the objectives l= 1, 2…..…j

f_t represent the transportation capacity of conveyance t.

e_{ijt}^{l} is transportation of a unit of the product from source i^{th} conveyance for the l^{th} represent transportation cost, delivery time, quantity of good, capacity, y_{ijt} represents the quantity to be transported from origin to destination by means of t^{th} conveyance.

3.6 Fuzzy multi-objective solid transportation problem:-
A fuzzy multi-objective solid transportation problem of minimizing objective function is:
Minimize:

$$Y_l \sum_{i=1}^{p} \sum_{j=1}^{n} \sum_{p=1}^{S} D_{ijp}^{-l}\ x_{ijt,}\ l= 1, 2\ldots\ldots\ldots J$$

Subject to the constraints:

$$\sum_{j=1}^{q} \sum_{t=1}^{S} y_{ijt} =\tilde{c}_i\ ,\ i= 1, 2\ldots\ldots\ldots p$$

$$\sum_{i=1}^{p} \sum_{t=1}^{S} y_{ijt}=\tilde{d}_j,\quad j = 1, 2\ldots\ldots\ldots q$$

$$\sum_{i=1}^{p} \sum_{j=1}^{q} y_{ijt} =\tilde{f}_t\ ,\ t= 1, 2\ldots\ldots\ldots\ldots S$$

$$y_{ijt} =\tilde{f}_t\ ,\ t = 1, 2\ldots\ldots S$$

$$y_{ijt} \geq 0,\ \forall_{ijt}$$

Where,

$\tilde{c}_i$ represents fuzzy supply of the product at i^{th}

$\tilde{b}_j$ be the fuzzy demand of the product at j^{th}

$\tilde{f}_t$ represents the fuzzy transportation capacity of conveyance t.

l represents the objectives l = 1, 2………. J

IV. CONCLUSION

Transportation problem is very important to solve decision-making. To find the salutation of transportation problem, a large amount of time and resource are needed. In order to handle the complexity of the problems and to provide decision associated to the problems, many researchers have exploited over the years in this area by using various traditional method and non-traditional method.

However, an optimization model of some transportation problem may habitually come across uncertain phenomena and so on due to the uncertainty, nondeterministic model parameters of transportation problems are precious by indistinctness and imprecision with linguistic terms, which are given by decision maker. In such conditions, multi-objective transportation problem. When traditional techniques are applied to solve such problems then this will give us an optimal solution. On the basis of literature, fuzzy transportation problem solution methodologies are different and for a simple real number or interval number based transportation problems solution approaches are different so it is necessary to find an approach with is applicable in both case.

Therefore, it is necessary for all such transport companies to decide some strategies according to their capacity before apply in such tendering procedure so that all companies will get maximum benefit.

V. FUTURE SCOPE

Developed model can be extend other multi-objective optimization problem, like: Assignment problem, transhipment problem, inventory model etc. linear and exponential membership function represent imprecise number for the solution of transportation problem.

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