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Study the Equations of Motion for a Hexacopter Aerial Vehicle based on The Newton-Euler Method

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ABSTRACT
This paper aimed at the design and investigation of a dynamic model of Unmanned Aerial Vehicle (UAV) Hexacopter in the 3-dimensional space. the derived equations of motion have been determined based on a Newton-Euler method. these equations were characterized as a nonlinear and coupled. Moreover, the aerodynamic effects and disturbances have been embedded into the model, in order to give a real motion to the Hexacopter. The Hexacopter is characterized as a VTOL (Vertical take-off landing) vehicle, in addition to hovering capabilities and agility so simply compared to the fixed-wing aerial vehicles. notwithstanding, it has a complicated dynamic model, such portrayed as an unstable and not able of translational motion without twisting about one of its axes. The concluded mathematical model had been carried out through the LabVIEW software besides control and simulation design module. consequently, the stability consideration had been analysed for multiple experimental states in order to demonstrate in advance the appropriate controllers for balancing and trajectory tracking.
Keywords :— UAV, Hexacopter Dynamics, Nonlinear Control, Coupled and Underactuated Models, Newton-Euler Method.

I. INTRODUCTION

This paper concentrates on the mathematical deriving of the equations of motion for the Hexacopter model in 3Dimensional space. further, modelling and simulation were included. the dynamical motion which concluded were defined as a highly nonlinear, multivariable and under-actuated system [1]. in addition, the Under-actuated systems, defined as a mechanical system in which the dimension of the configuration space exceeds that of the control input space, that is, with fewer control inputs than degrees of freedom [2]. Modelling of such a system is not a little problem because of the coupled variables of the Hexacopter [3]. The contributions of this research are obtaining an exact and detailed mathematical model of a Hexacopter UAV and investigating the problems of nonlinearity and coupled parameters in hovering scenarios. We invented the equation of motion of the whole system using the Newton-Euler formulation for translational and rotational dynamics of a rigid body [4, 5, 6]. The disturbances were presented as an unrestricted environment and aerodynamic effects for simulation, which were not mentioned in most of the literatures.

II. REFERENCE FRAMES FOR THE UAV MICRO-COPTER

This section describes the kinematics of the UAV hexa-copter. The structure of the hexa-copter and the rotational directions are illustrated in Figure 1. Only two reference frames are necessary: earth inertial frame (E-frame) and body-fixed frame (B-frame). The motion is planned by using geographical maps, with North, East and Down (NED) coordinates [1, 6]. This earth fixed frame is seen as an inertial frame in which the absolute linear position (x, y, z) of the hexa-copter is defined. The mobile frame (XB, YB, ZB) is the body fixed frame that is centred in the hexa-copter centre of gravity (CG) and oriented as shown in Figure 1. The angular position of the body frame with respect to the inertial one is defined by Euler angles: roll Ø, pitch ϴ and yaw ***Ψ***. These together form the vector: $\sigma = [\text{Ø}\ \Theta\ \Psi]^T$

$\phi\ and\ \theta \in]-\frac{\pi}{2}, \frac{\pi}{2}[;\ \Psi \in]-\pi, \pi[$.

The inertial frame position of the vehicle is given by vector $\xi = [x\ y\ z]^T$ [7, 5, 8]. The transformation from the body frame to the inertial frame is realized by using the well-known rotation matrix C_b^n defined in [7,8]. Which is orthogonal and $C_b^{n^T} = C_b^{n^{-1}} = C_n^b$. In addition, the transformation matrix for angular velocities from the body frame to the inertial one is S, mentioned in [9]. Where $\dot{\sigma} = S\ .\ \Omega$, $\dot{\xi} = C_b^n\ .\ V$, the angular velocity Ω is defined by the vector $\Omega = [p\ q\ r]^T$, and the linear velocity is defined by the vector $V = [u\ v\ w]^T$ in the body frame. It is important to observe that S can be defined if and only if $\theta \neq \frac{\pi}{2} + \pi k\text{: } k \in Z$. This is the main effect of Euler's formulation that leads to the gimbal lock, typical situation in which a degree of freedom is lost [10, 11] and this is not our concern here.

III. AERODYNAMIC FORCES AND MOMENTS IN AXIAL FLIGHT

The UAV Hexacopter systems are quite complex, their movements are governed by several effects either mechanical or aerodynamic. Our aim is to provide the mathematical equations driving the dynamical behaviour of the hexa-copter by means of a generalization of the Quadcopter model presented in [9].

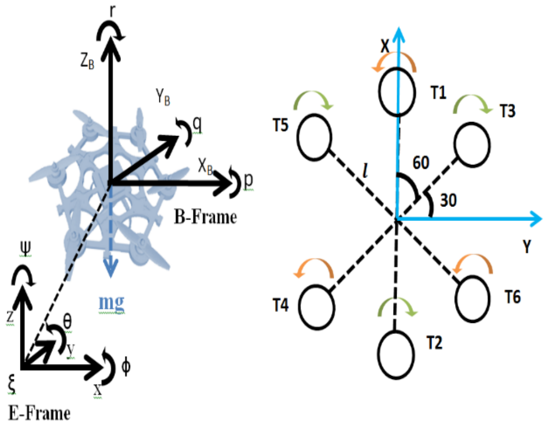


Fig 1. UAV Hexacopter structure and Frames.

The motion of a rigid body can be decomposed into the translational and rotational components. The Newton-Euler equations are used. In order to modelling, the assumptions have been made that the hexa-copter is a rigid body and has a symmetrical structure. Therefore, the following equations are obtained:

$$\begin{bmatrix} mI_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & J \end{bmatrix}\begin{bmatrix} \dot{V} \\ \dot{\Omega} \end{bmatrix}_B + \begin{bmatrix} \Omega\times(mV) \\ \Omega\times(J\Omega) \end{bmatrix}_B = \begin{bmatrix} \sum F \\ \sum M \end{bmatrix}_B$$

Where $I_{3\times3}$ an identity matrix size 3×3, $0_{3\times3}$ a zero-matrix size 3×3, m Є R is the total mass of Hexacopter and J Є R^3 the diagonal inertia matrix.

IV. FORCE ANALYSIS

A. **Thrust Force:** The main force affecting the aircraft movement is the thrust force resulting from the motors and propellers that lift the aircraft in the air. The model consists of 6 motors, and according to the suggested engineering model (fig 1), the motors in the model are parallel and perpendicular to the aircraft surface, so we conclude that the total thrust force vector of the aircraft is T and it is the sum of the propellers thrust force vectors $\sum_{i=1}^{6} T_i$, Then thrust and torque, are [4]:

$T = \rho C_T A R^2 \omega^2$, and $Q = \rho C_Q A R^3 \omega^2$

Where CT and CQ are respectively thrust and torque coefficients, ρ is the air density and A the disc area. The thrust and torque coefficients can be written as:

$$C_T = \tfrac{1}{4}\sigma C_{L\alpha}\left[\tfrac{2\theta_b}{3} - (\gamma_c + \gamma_i)\right],$$

$$C_Q = \tfrac{1}{2}\sigma\left[C_{L\alpha}(\gamma_c + \gamma_i)\left\{\frac{\theta_b}{3} - \frac{\gamma_c + \gamma_i}{2}\right\} + \frac{C_D}{4}\right.$$

Where, σ is the rotor solidity, $C_{L\alpha}$ is the lift slope coefficient, C_D is the drag coefficient. γ_c, and γ_i are the inflow factors. Finally, the total force of thrust generated by the six propellers is defined as: $F_{thrust} = [0 \quad 0 \quad \sum_{i=1}^{6}|T_i|]^T$.

B. ***Drag Force:*** It is the opposing force to the traveling of the solid body in air resulted from the aerodynamic friction [9] and can be expressed it at the body's frame: $F_{aero} = K_T.V$; where K_T is a diagonal matrix related to the aerodynamic friction constant by the parameter k_t [8,4].

C. ***Gravitational Force:***

The gravity force is directed toward the centre of earth, therefore, it is possible to describe the relation of gravity force within the frame of the aircraft body by the following equation [8], where g is gravity constant:

$$F_{grav} = m.C_n^b.\begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} = m.g.\begin{bmatrix} -sin\theta \\ sin\emptyset.cos\theta \\ cos\emptyset.cos\theta \end{bmatrix},$$

D. ***Disturbance Force:*** Other forces like the Coriolis force from the earth [8], the wind and Euler forces are considered as a disturbance, summarized as:

$F_{dist} = [F_{d1} \quad F_{d2} \quad F_{d3}]^T$,

Therefore, the equations of motion that govern the translational motion based on Newton-Euler formulation with respect to the body frame are:

$$\sum F = F_{thrust} - F_{aero} - F_{grav} + F_{dist} = m.\dot{V} + \Omega\times(m.V),$$

$$\begin{cases} \dot{u} = -\frac{k_t}{m}u + g.sin\theta - (qw - vr) + \frac{Fd1}{m} \\ \dot{v} = -\frac{k_t}{m}v - g.sin\emptyset.cos\theta - (ru - pw) + \frac{Fd2}{m} \\ \dot{w} = \sum_{i=1}^{6}|T_i| - \frac{k_t}{m}w - g.cos\emptyset.cos\theta - (pv - qu) + \frac{Fd3}{m} \end{cases}$$

Then the equations of motion with respect to the inertial (Earth) frame are:

$$\ddot{\xi} = [\ddot{x} \quad \ddot{y} \quad \ddot{z}]_E^T = C_b^n.[\dot{u} \quad \dot{v} \quad \dot{w}]_B^T$$

V. MOMENTS ANALYSIS

Suppose, the inertia matrix of the aircraft is J, the structure of the aircraft is symmetric, and the controllers and the load are in the centre, we conclude that the inertia matrix is of the following form:

$$J = \begin{bmatrix} J_{xx} & 0 & 0 \\ 0 & J_{yy} & 0 \\ 0 & 0 & J_{zz} \end{bmatrix}; J \in R_{3\times3}$$

The moments acting on the centre of the aircraft can be analysed as follows:

A. ***Propeller Moments:***

The M_{thrust} is a part of the external moments acting on the system, described by the propeller thrust $\sum_{i=1}^{6} T_i$ generated by propellers, and the distance l from CG to the center of the propeller. The attitude of the vehicle in the air, i.e., Euler angles $\sigma = [\phi\ \theta\ \psi]^T$ change, by controlling the angular velocity of engines, i.e., this moment can be expressed as

$M_{thrust} = [M_x \quad M_y \quad M_z]^T$, Where M_x, M_y, M_z are the moments about the axes X_B, Y_B, Z_B in the body frame, noticing that the torque vectors direction as in Figure (2) [8,1,5]. The moments as follows:

$$M_{thrust} = \begin{bmatrix} \frac{\sqrt{3}}{2} l(|T_3| - |T_4| - |T_5| + |T_6|) \\ \frac{1}{2} l(|T_3| - |T_4| + |T_5| - |T_6| + 2|T_1| - 2|T_2|) \\ \rho C_Q A R^3(\omega_1^2 + \omega_4^2 + \omega_6^2 - \omega_2^2 - \omega_3^2 - \omega_5^2) \end{bmatrix}$$

A. The aerodynamic moment:

It is the moment resulting from the aerodynamic friction in, and it affects negatively the total moment. The aerodynamic moment is expressed: $M_{aero} = K_R . \Omega$, where K_R is a diagonal matrix related to the rotational aerodynamic friction constant by the parameter K_r [8,4].

c. Disturbance moment: It is the total of the disturbances affecting the torque around the aircraft axes resulting from motors disturbances, the wind, and the load.

$$M_{dist} = [M_{d1} \quad M_{d2} \quad M_{d3}]^T ,$$

Therefore, the equations of motion with respect to the body frame are:

$$\sum M = M_{thrust} - M_{aero} + M_{dist} = J.\dot{\Omega} + \Omega \times (m.\Omega)$$

$$\dot{p} = \frac{\sqrt{3}}{2Jx} l(|T_3| - |T_4| - |T_5| + |T_6|) - \frac{k_r}{Jx} p - qr\frac{(Jz-Jy)}{Jx} + \frac{M_{d1}}{Jx}$$

$$\dot{q} = \frac{1}{2Jy} l(|T_3| - |T_4| + |T_5| - |T_6| + 2|T_1| - 2|T_2|) - \frac{k_r}{Jy} q - pr\frac{(Jx-Jz)}{Jy} + \frac{M_{d2}}{Jy}$$

$$\dot{r} = \frac{\rho C_Q A R^3}{Jz}(\omega_1^2 + \omega_4^2 + \omega_6^2 - \omega_2^2 - \omega_3^2 - \omega_5^2) - \frac{k_r}{Jz} r - pq\frac{(Jy-Jx)}{Jz} + \frac{M_{d3}}{Jz}$$

The equations of motion that govern the rotational motion for the Hexacopter with respect to the inertial (Earth) frame are: $\ddot{\sigma} = [\ddot{\phi} \quad \ddot{\theta} \quad \ddot{\psi}]_E^T = S.[\dot{p} \quad \dot{q} \quad \dot{r}]_B^T$. The suggested mathematical model as shown in the equations is characterized by nonlinearity, time variance, and coupled variables where Figure (2) shows a block diagram that contains the input and output variables of the aircraft dynamic model.

VI. CONTROL MECHANISM

The block diagram in Figure (2) shows the variables of the mathematical model of a Hexacopter aircraft. The control mechanism is through controlling the motors speed variables $\omega_1, \omega_2 \ldots \omega_6$ by a defined style explained as follows:

1- There are 3 movements that describe all possible combinations of attitude: Roll (rotation around the X axis by angle ϕ), Pitch (rotation around the Y axis by angle θ), and Yaw (rotation around the Z axis by angle ψ). The roll control is obtained by changing the velocity of motors 3, 4, 5, 6, and this movement is called lateral motion. Then the pitch control is obtained by changing the velocity of all motors, resulting in the longitudinal motion. Finally, the yaw control is obtained by changing the velocity of all motors.

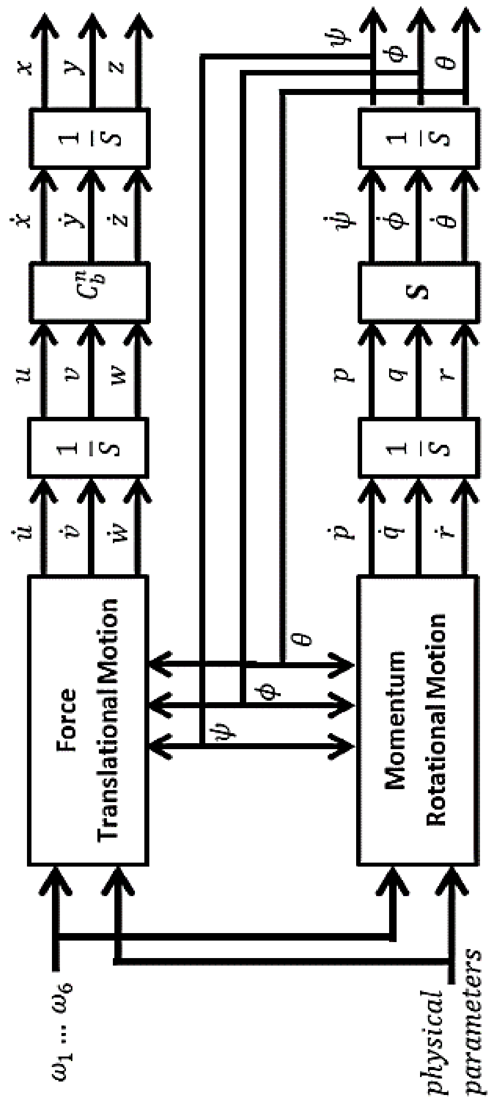


Fig. 2 Mathematical model of Hexacopter.

2- The change of motors speed for attitude control should be fixed and based on differential control strategy as seen in Figure (1) and equations of moments, i.e., the Pitch control around axis Y is obtained by changing the torques around this axis by increasing (T1, T3, T5) and decreasing the another side (T2, T4, T6) using the following equation: $\ell * \cos 60 * (|T_3| - |T_4| + |T_5| - |T_6|) + |T_1| - |T_2|$ And the roll control is obtained by: $\ell * \cos 30 * (|T_3| - |T_4| - |T_5| + |T_6|)$ While the yaw control is based on the torque difference between the neighboring motors: $Q_1 + Q_4 + Q_6 - Q_2 - Q_3 - Q_5$

3- Altitude control is obtained by changing all motors' velocity with a fixed change. This is based on the force equations in Z component, noticing that the thrust is equivalent to the square of the motors angular velocities. To increase the altitude, all motors velocities must be increased, and vice versa. The equation that governs the altitude is: $altitude\_control \Leftrightarrow \sum_{i=1}^{6} |T_i|$

From the control problem based on Figure (2) and its control equations, which govern the attitude and altitude in space, the artificial vector $U = [u_p \; u_r \; u_y \; u_T]^T$ can be found. This simplifies the control of the system in Pitch, Roll, Yaw and altitude movements instead of using real motors' velocities vector $\omega = [\omega_1 \; \omega_2 \; \omega_3 \; \omega_4 \; \omega_5 \; \omega_6]^T$ [5]. Now we can put the equations that connect between artificial and real input vectors as follows:

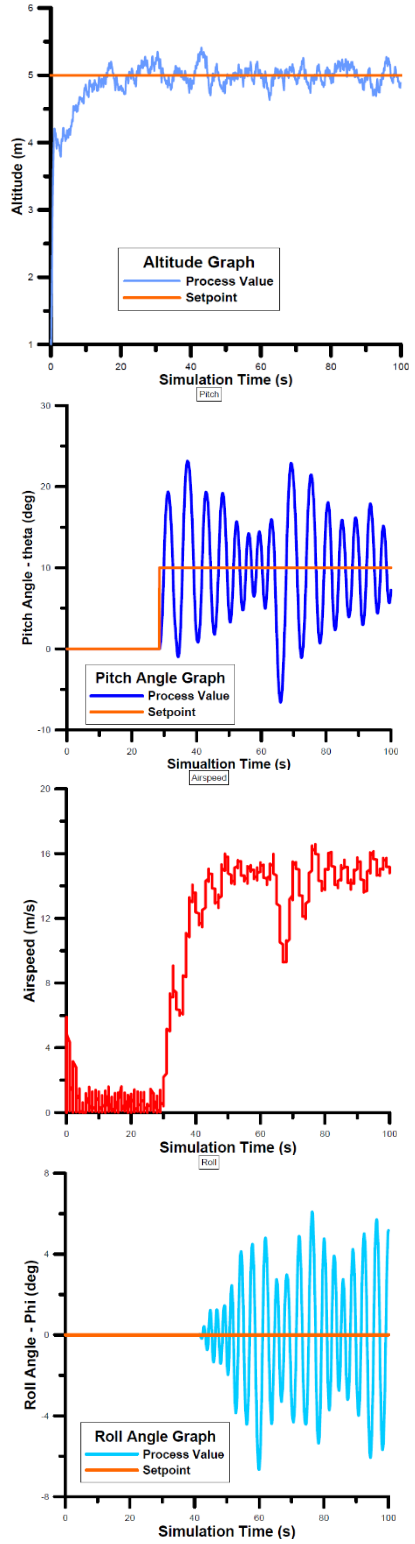
$$\begin{cases} \omega_1 = u_T + u_p + u_y \\ \omega_2 = u_T - u_p - u_y \\ \omega_3 = u_T + 0.5 * u_p - 0.866 * u_r - u_y \\ \omega_4 = u_T - 0.5 * u_p + 0.866 * u_r + u_y \\ \omega_5 = u_T + 0.5 * u_p + 0.866 * u_r - u_y \\ \omega_6 = u_T - 0.5 * u_p - 0.866 * u_r + u_y \end{cases}$$

VII. CONCLUSIONS

The UAV Hexacopter model has been completely designed and conducted using LabVIEW software for the simulation and stability study. The system parameters were used in the model are listed in table 1. In order to study the coupled and underactuated problems of our model, an application is conducted by simulation using Runge-Kutta 2 method with fixed step 0.05 (sec). We analysed the coupled matter between all parameters of the Hexacopter in figure (3) when the input up set to 10 to control the pitch angle and remaining control inputs were fixed as ($ur = uy = 0, ut = 1230 \; rpm$). This scenario is flight hovering at altitude 5 meters in the air. In this paper, we developed a real dynamic model addressing the nonlinear, time-variant and underactuated problems, because of the complex dynamics. This requires precise trajectory control to stabilize the whole system and drive the Hexacopter to the desired trajectory of Cartesian position, attitude and airspeed. In future optimal control will be used to investigate the stability in addition to studying those characteristics between the variables of the equations of motion.

TABLE 1. PARAMETERS USED IN THE SIMULATION

m=4 kg	**g=9.806 m/s²**	**l=0.36 m**	**ρ=1.293 kg/m³**	**R =0.15 m**
J_x, J_y=3.8e⁻³ N.m.s²/rad	**A =0.071 m²**	**k_t=4.8e⁻² N.s/m**	**k_r=6.4e⁻⁴ N.m.s/rad**	
J_z=7.1e⁻³ N.m.s²/rad	**C_Q=1.037e⁻³**	**C_T=0.01458**		



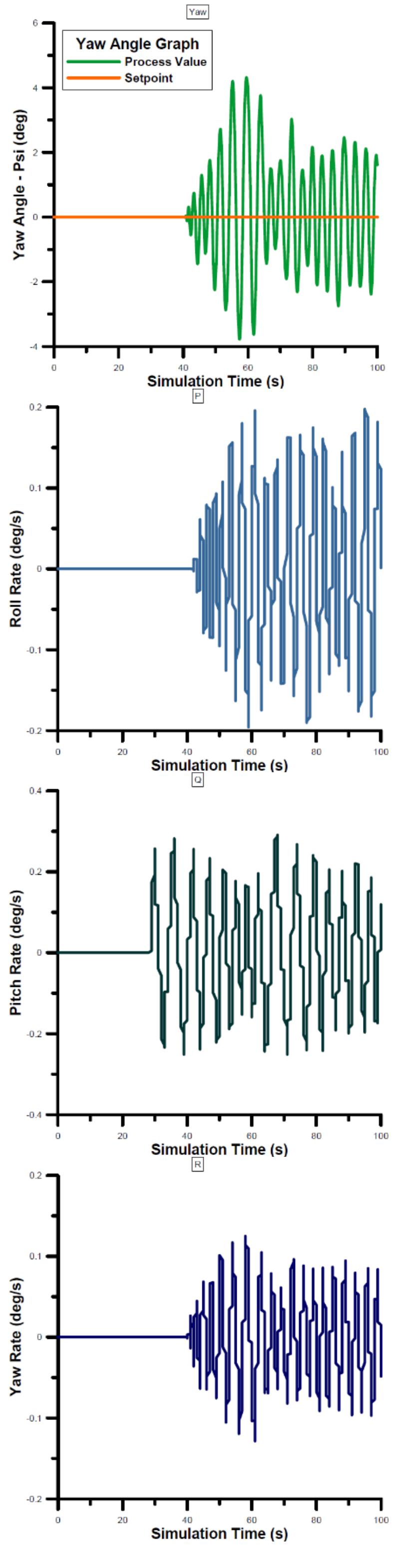


Fig. 3 Change and coupling of the Hexacopter parameter Due to input change of up

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